How Predictable Are Equity Covariance Matrices?

Evidence From High Frequency Data For Four Markets

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Abstract

Most pricing and hedging models rely on the long run temporal stability of a sample covariance matrix. Using a large data set of equity prices from four countries, the US, UK, Japan and Germany, we test the stability of realized sample covariance matrices using two complementary approaches: a standard covariance equality test and a novel matrix loss function approach. Our results present a pessimistic outlook for equilibrium models that require the covariance of assets returns to mean revert in the long run. We find that whilst a daily first order Wishart autoregression is the best covariance matrix generating candidate, this non-mean reverting process cannot capture all of the time series variation in the covariance generating process.

Keywords: Realized Covariance, Microstructure, Wishart Distribution
JEL Classification: G14, G15, G17

1. Introduction

The extent to which the covariance matrix describing the second moment of asset returns remains stable, in the long run, is a fundamental question and a property that has a major influence on the effectiveness of portfolio analysis and modeling of asset prices. Standard asset pricing models such as the CAPM and APT are founded on the assumption that the covariance matrices between assets, and between assets and factors, are time invariant.

In the dynamic covariance context, such models can still operate with short run fluctuations and non-persistence in the covariance matrix as long as the long run stability exists. For instance, all multivariate ARCH representations converge to an equilibrium covariance matrix in the long run.

Several studies, such as Bollerslev et al. (1994) and Hansen and Lunde (2006) have found that volatility (as measured by realised standard deviations) is highly persistent in the long run, postulating that the volatility process itself is either a unit root or near unit root. It is difficult to overstate the importance of rejecting long-run mean reversion in the variance-covariance process to the finance industry and the method by which most companies manage and trade risk. Williams and Ioannidis (2011), for instance, show that for n assets, a hedging strategy would need $n(n+1)/2$ extra hedging instruments for full delta neutrality in the presence of a covariance process driven by a matrix Brownian motion.

*Corresponding Author. Telephone +44 191 33 45301. This paper represents an extension to prior work by the authors on discrete time volatility models where the underlying asset returns are driven by a Wishart autoregression and the subsequent implications for the forecasting of covariance risk and asset pricing. We would like to thank Angela Black, Christos Ioannidis, Yuzhi Cai, Alan Hawkes, George Kapetanios for comments and Joe Swierzbinski for their continuous and constructive advice. The working draft of this paper was originally entitled "Testing the Stability of Realized Covariance Matrices". We would also like to thank two anonymous referees for helpful comments and the editor Derek Bunn. Williams recognises the support of the Technology Strategy Board project 'Cloud Stewardship Economics' and HP Labs Bristol for advice and training on large dataset handling.

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There is plenty of empirical evidence to suggest large shocks, such as the global financial crisis since late 2006 or the terrorist attack on the US on September 11, 2001, can result in permanent structural changes in the correlation and volatility between assets (see Bloom (2009)). However, for large highly diversified multinational companies, such as the 30 public companies comprising the Dow Jones Industrial stock market index, the covariance structure would be expected to be relatively stable, hence mean reverting, over longer periods of time, given that their underlying dividend processes should be from a relatively small number of principal components.

Designing statistical methods that distinguish between different covariance generating processes (CGP) is therefore important for a wide range of financial activities. For instance, the use of equally weighted portfolios may not be inferior to optimised portfolios as the optimised weights are computed from parameters which are both stochastic and possibly non-mean reverting. The pricing of multi-asset derivatives such as relax certificates, which place floors on individual stocks and combinations of stocks, must incorporate the covariance risk directly into the pricing kernel.

Most classical financial theories rely on the asymptotic stability of the long run sample covariance matrix of asset returns There is empirical evidence, mostly estimated on daily or lower frequency data, has indicated that the covariance structure of assets changes substantially through time. The issues are whether these fluctuations in the covariance structure are mean reverting or whether the forward looking covariance structure is inherently uncertain. Further, identification of the level of variation in the covariance structure relative to a long run benchmark allows us to identify whether standard equilibrium pricing models are viable for certain types of mid to long term investors.

Traditional classical theories can still possibly be adapted to a time varying covariance framework, with little loss of generality, if the conditional covariance matrix appears mean reverting in the long run (see, for instance, Heston (1993)). However, in this paper, we present substantial and significant evidence to suggest that the covariance matrices for a broad collection of US, UK, Japanese and German equity returns do not exhibit any form of long run stability. The results suggest that even a first order Wishart autoregressive process does not fully capture the variation in the covariance structure of minute returns. Thus, the assumption of the stability of the covariance matrix appears to be questionable.

To address the issue of identifying whether the covariance matrix is stable, we utilise three recent contributions to the literature. First, we propose new robust measures of realised covariance from high frequency data by implementing the realized kernel methodology outlined in Barndorff-Nielsen et al. (2009a, 2008) to provide solutions to the selection of the appropriate covariance matrix when constructing hedged portfolio. This measure is less affected by the microstructure noise inherent in high frequency data.

Second, this research, for the first time, directly links estimation of the covariance generating process (CGP) to the covariance equality test literature to develop a robust diagnostic test of covariance stability. Specifically, we set covariance equality to be the null hypothesis. This means rejection of stability will be considered as a type II error rather than a type I error. Thus, our hypothesis, instead of looking to find instability, is seeking to reject stability. To achieve this, we propose a time varying loss function approach with a one year rolling window to mitigate the possibility of transient outliers affecting the test. This extends the current literature by proposing a time-varying unit root test of the covariance matrix to reveal time varying features of the CGP.

Further, this large scale empirical study applies the latest development from the literature on forecast breakdowns to a latent variable analysis. Prior research, for instance Zhang et al. (2005), has focused on a fixed sampling frequency and consistency relative to simulation benchmarks. The modeling in this study uses both a fixed sampling frequency (minute) and a systematically-varying time horizon (daily, weekly, monthly and quarterly) to provide enhanced forecast stability through both in-sample and out-of-sample comparisons.

In addition, comparison tests of out-of-sample forecasts are constructed for realised covariance matrices using long run ex-post and short run ex-ante CGPs and are compared using a novel comparative loss function approach.

Moreover, a Monte Carlo approach is applied to compute a rolling test statistic and corresponding confidence bounds. Inference is taken from a combination of Monte-Carlo simulation, bootstrap re-sampling and standard asymptotic theory. Our results generally suggest that neither of the simplest descriptions of the covariance are wholly adequate and it is evident that more complex models are needed to correctly capture the covariance dynamics of asset returns.

In this paper, a cross-country comparison of covariance stability is undertaken using data for actively traded stocks from the US (New York Stock Exchange - NYSE), UK (London Stock Exchange - LSE), Japanese (Tokyo Stock Exchange - TSE) and German (Deutsch Borse Group, Frankfurter Wertpapierbörse/Frankfurt Stock Exchange - FSE) equity markets. A large data set of minute frequency sampled equity returns is utilised to compute covariance matrices from daily, weekly, monthly and quarterly blocks; and compare them, in sample, to a long run (yearly and whole sample of 14 years) covariance matrix using an equality test based on the properties of the Wishart distribution. This, to the authors' knowledge, is the first paper to model such large scale cross markets of stocks over long sample periods. In total, fourteen years of data are utilised at the minute as the highest frequency for sampling 94 stocks in these four markets.

In summary, this paper is to outline a simple framework to assist in identification of the CGP and test it on a large ultra-high frequency dataset over a long time period for broad classes of equities crossing four major stock markets. The computed test statistics generally reject the null of equality of the short run daily, weekly and monthly high frequency sample groups versus the long run, yearly and whole sample sets. Interestingly, the results suggest that the short run covariance matricies do not appear to improve on the long run covariance matrices in forecasting day by day risk. This finding suggests that the covariance structure exhibits high degrees of variation and that models with explicit non-convergent stochastic covariance dynamics, such as the Williams and Ioannidis (2011) or the Buraschi et al. (2010) models, should be recommended to measure risk ahead of traditional long run constant covariance models or mean reverting stochastic covariance models.

The paper is organised as follows: §(2) briefly reviews some of the related literature; §(3)outlines the testing approach and describes the two broad types of stability test that are implemented; §(4) covers the extensive data processing needed for high frequency research; §(5) briefly reviews the empirical analysis and results, whilst §(6) concludes. An internet appendix with the asymptotic form of the Wishart distribution estimated via a heteroskedasticity and autocorrelation consistent (HAC) kernel estimator is available from the authors.

2. Related Literature

Whilst univariate high frequency realised variance analysis has a relatively long history, the computational issues associated with the multivariate context is a more recent development in the financial econometrics literature. Ledoit and Wolf (2004, 2003) offer comprehensive reviews of modern techniques for computing well-conditioned covariance matrices of asset returns. More recent work by Barndorff-Nielsen and Shephard (2004) and Barndorff-Nielsen et al. (2009a, 2008) has focused on the sampling and consistency of realised covariance estimators over a variety of time-scales ranging from tens of minutes to one second in frequency. The choice of sampling scale is a key feature of realised covariance estimation. One of the major issues with estimating covariance matrices is that for most common uses, regression analysis and portfolio selection included, the covariance matrix must be invertible and, therefore, well conditioned. In low dimensional settings, this invertibility requirement becomes a more acute and common problem in financial econometrics with cross sections.

Recent work by Kyj et al. (2010) has shown a variety of covariance models sampled at five minute intervals and demonstrated their performance in actively managed portfolios targeting the global minimum variance portfolio. In contrast to Kyj et al. (2010), however, our study implements a one minute sampling frequency to estimate daily covariances, to ensure that the sampling interval is high enough to fulfil the criteria of $m > 10n$ as suggested by Ledoit and Wolf (2003) [1]. The consequent issue that as the sampling frequency increases, micro-structure effects can contaminate the covariance estimator, is discussed, for example, in

[1] Ledoit and Wolf (2003) suggest that if the data generating process is assumed to be conditionally multivariate-Gaussian or fairly closely related, sampling at a frequency that results in a conditional data matrix with rows m approximately ten times more than a system with the dimensionality of n is more desirable as a well conditioned covariance matrix.

Bandi and Russell (2006). As such, we propose the simple linear covariance estimator needs to be replaced with a heteroskedasticity and autocorrelation consistent (HAC) alternative.

By contrast to the recent realised covariance literature, there is substantial research on comparisons between competing forecast models. The Diebold and Mariano (1995) approach on competing forecasts offers a good summary of the properties of these comparative approaches. More recently, Giacomini and White (2006) and Giacomini and Rossi (2009) have outlined various bootstrap methods for assessing predictive accuracy and enhancing robustness checks to the significance of these comparisons.

Another issue of interest is to compute comparative loss functions on the second moments when the conditional variance-covariance matrix is measured with error. Patton (2006) discusses this issue with respect to univariate forecasts of variance, suggesting a series of robust loss functions, and it is the multivariate extension of these that motivates the second part of the analysis in this paper.

3. Methodology

Two methods for testing the stability of the conditional covariance matrix relative to a longer run benchmark are implemented in this paper. First, in §§(3.1) an 'in sample' matrix equality test based around the Wishart matrix distribution is demonstrated. Second, in §§(3.3) a matrix loss function approach is derived for comparing the fit of alternative forecasting models. Some further results on the asymptotic distributions of the test statistic are available in an internet appendix.

For n assets, the notation is as follows, $y_{i,t} = [y_{1,i,t}, \dots, y_{n,i,t}]'$ is the vector of log prices, where $t \in \{1, \dots, T\}$ is the index of the assets by day and $i \in \{1, \dots, m+1\}$ is the index of intraday prices. For simplicity of initial notation assume that $m_{t=1} = m_{t=2} = \dots = m_{t=T-1} = m_T$: therefore the total number of observations is mT. The $m \times n$ matrix of intraday centred returns $X_t = [x_{1,t}, x_{2,t}, \dots, x_{m,t}]'$ is formed by filtering the log differences of the prices $\Delta y_{i,t} = y_{i+1,t} - y_{i,t}$ by the ex-post daily mean, $x_{i,t} = \Delta y_{i,t} - \mu_t$, where $\mu_t = m^{-1} \sum_{i=1}^{m} \Delta y_{i,t}$. The returns are filtered in this way as the object of interest is the variation and covariation of the returns and not the direction. Next consider an index j which represents the daily, weekly or monthly collections of filtered return matrices X_t. Setting $X = [X_1', \dots, X_T']'$, where X is the $mT \times n$ matrix of all data, $X_j = [X_{t+1}', \dots, X_{t+h}']_j'$ for appropriate t to be the collection of daily $h = 1$, weekly $h \simeq 5$, monthly $h \simeq 22$ and quarterly $h \simeq 67$ data matrices[2]. As such, there are $J = Th^{-1}$ collections of daily data matrices over the sample T.

Conditional correlation matrices R_j are computed from the decomposition $\hat{\Sigma}_j = R_j \cdot h_j h_j'$, where h_j is element by element square root of the diagonals of the $h_j^2 = diag \hat{\Sigma}_j$ and $\cdot$ is the Hadamard element by element product. Using the notation of Muirhead (1983), the total quadratic variation is denoted $A = X'X$ and the j^{th} time indexed quadratic variation is denoted as $A_j = X_j' X_j$. The j^{th} indexed sample covariance matrix is denoted $\hat{\Sigma}_j = (mh)^{-1} A_j$. Gourieroux et al. (2009) outline a multivariate stochastic volatility model driven by a Wishart progression. Following from this innovation this paper seeks to discriminate between two potential candidate processes

$$\begin{array}{ll} A_j \sim \mathcal{W}\left(mh, n, \Sigma\right) & \text{Long run ex-post} \\ A_j \sim \mathcal{W}\left(mh, n, (mh)^{-1} A_{j-1}\right) & \text{Short run ex-ante} \end{array}$$

The second CGP can be rewritten as $A_j \sim \mathcal{W}(mh, n, \hat{\Sigma}_{j-1})$, i.e. a first order Wishart Autoregression. The degree of variation is dictated by the row-wise degrees of freedom mh, assuming that the covariance over a single day is stable, then the degree of stochastic covariation is entirely dictated by h. The analysis is split into two sections, first to establish whether there is covariance equality between $\Sigma_{j=1} = \Sigma_{j=2} =, \dots, = \Sigma$. Second, an estimate $\hat{\Sigma} = (mT)^{-1} X'X$ of Σ is used as a forecast benchmark and run comparative forecasts against $\hat{\Sigma}_j$. These two steps are outlined in §§(3.1) and §§(3.3) respectively.

[2]Weekly, monthly and quarterly collections are approximate $\simeq$ is used as national holidays are excluded.

3.1. Static Covariance Equality Tests

The covariance equality statistic of the null hypothesis $\Sigma_{j=1} = \Sigma_{j=2} =, \ldots, = \Sigma$ is defined as,

$$\Lambda^* = \frac{\det(\hat{\Sigma})^{\frac{mT}{2}}}{\prod_{j=1}^{J} (\det \hat{\Sigma}_j)^{\frac{mh}{2}}} \tag{1}$$

If $X \sim \mathcal{N}(0, \Sigma)$, Muirhead (1983) shows that $2 \log \Lambda^* \sim \chi^2(\frac{1}{2} n (n+1)(J-1))$, therefore for a given critical value c_α, the null hypothesis of $\Sigma_{j \in \{1,\ldots,J\}} = \Sigma$ is rejected for $\Lambda^* \geq c_\alpha$. The proof of the asymptotic distribution of Λ^* is well known and the interested reader is directed to Muirhead (1983) or Ledoit and Wolf (2002) for Monte-Carlo evidence for large cross sections. All tests in this paper are described in terms of level of significance in rejecting the null (e.g. 10, 5, 1%) and use the term confidence level to denote whether a rolling test statistic is above a certain significance threshold. All test and critical statistics are divided by a constant, in this case, 10e6 following the procedure suggested in Muirhead (1983).

3.2. Rolling Variation Tests and Resampling

Financial data, particularly at very high frequencies, exhibits high degrees of autocorrelation and heteroskedasticity and as such the covariance estimator needs to be corrected for these contaminants. A variety of HAC covariance estimators are available and various choices can inject some considerable variation in the resultant covariance matrices. Barndorff-Nielsen et al. (2008) conduct a variety of Monte Carlo experiments to test for the best unbiased kernel suggesting that estimate for the covariance on day t

$$\tilde{\Sigma}_t(\tau) = \frac{1}{m} \sum_{r=-\tau}^{r=\tau} K\left(\frac{r}{\tau+1}\right) \tilde{\Gamma}_r, \tilde{\Gamma}_r = \left\{ \begin{array}{l} \sum_{i=|r+1|+1}^{m} x_{i,t}x'_{i-r,t}, r \geq 0 \\ \sum_{i=|r+1|+1}^{m} x_{i+r,t}x'_{i,t}, r < 0 \end{array} \right.$$

computed with a Parzen Kernel is the best performer. It is stated in a normalised time such that for a variable z, the functional form is

$$K(z) = \left\{ \begin{array}{ll} 1 - 6z^2 + 6z^3 & 0 \leqslant z \leqslant \frac{1}{2} \\ 2(1-z)^3 & \frac{1}{2} \leqslant z \leqslant 1 \\ 0 & z > 1 \end{array} \right. \tag{2}$$

To test for the optimal bandwidth parameter a Frobenius matrix norm convergence criteria is implemented, such that $b(\tilde{\tau} - \tau) = \|\tilde{\Sigma}_t(\tilde{\tau}) - \tilde{\Sigma}_t(\tau)\|_F$, where $\tilde{\tau} > \tau$. The optimal bandwidth is chosen when $\Delta b(\tilde{\tau} - \tau)$ is monotonic.

The object of interest is to test the variation of the local short run covariance matrix $\hat{\Sigma}_j$ in relation to an ex-post estimated long run covariance matrix $\hat{\Sigma}$. The choice of time periods is arbitrary and primarily driven by data availability and an attempt to find investment horizons compatible with standard investment management strategies. The system is resampled by choosing random start days within the first 6 month period to account for starting time effects and take an average loss (averaging across the confidence bounds is also implemented). In effect the rolling loss functions allow the econometrician to identify periods when one particular forecast model dominates another, for a given choice of loss function that comparatively penalises a lack of fit.

3.3. Matrix Loss Function

The second approach for testing the stability of the ex-post realised covariation of asset returns is to utilise a matrix loss function approach along the lines of the Diebold and Mariano (1995). Consider two competing forecasts $\Sigma_t^g \in \mathbb{C}^{n\times n}$ and $\Sigma_t^h \in \mathbb{C}^{n\times n}$, for example the long run and short run forecasts described in the previous section, where $\mathbb{C}^{n\times n}$ is the field of all positive definite covariance matrices. For a function, $f : (\mathbb{C}^{n\times n}, \mathbb{C}^{n\times n}) \to \mathbb{R}_+$, the comparative loss for the current day is

$$\ell_t(\theta) = f\left(\Sigma_t^g, \tilde{\Sigma}_t \,|\theta\right) - f\left(\Sigma_t^h, \tilde{Sigma}_t \,|\theta\right) \tag{3}$$

Following Diebold and Mariano (1995) the heteroskedasticity and autocorrelation consistent normalised moving average sum of this forecast is denoted as

$$L_t(\theta) = N^{-1} \sum_{i=0}^{N} \ell_{t-i}(\theta) \left(\sum_{i,j=0}^{N} k_{|i-j|} \ell_{t-i}(\theta) \ell_{t-j}(\theta) \right)^{-\frac{1}{2}} \tag{4}$$

where θ is the parameter vector of the function $f(\cdot)$, $k_{i,j}$ is a kernel weight and N is again the size of the daily moving average window. In keeping with convention, Bartlett weights $k_{|i-j|} = 1 - \frac{|i-j|}{N}$ are used to construct the HAC variance of the mean comparative loss. For comparative forecast evaluations, the test statistic for the Diebold and Mariano (1995) approach is assumed to be $\mathcal{N}(0,1)$. To our knowledge, the current statistical literature on matrix functions of random matrices has not yet proved the generality of this distributional assumption, so a Monte-Carlo approach is implemented to derive the confidence bounds.

The Monte-Carlo algorithm proceeds as follows, first before the time loop draw pairs of $m \times n$ random matrices, $A^g_{i\in\{1,\ldots,B\}}$ and $A^h_{i\in\{1,\ldots,B\}}$, consisting of independent and identically, $B = 1,000$, distributed $\mathcal{N}(0,1)$ random numbers. At each time step compute the alternative covariance matrix forecasts Σ^g_t and Σ^h_t and compute their Cholesky factorized counterparts Q^g_t and Q^h_t. Next, construct sets of candidate covariance matrices $\tilde{\Sigma}^g_{i,t} = Q^g_t A^g_i A'^g_i Q'^g_t$ and $\tilde{\Sigma}^h_{i,t} = Q^h_t A^h_i A'^h_i Q'^h_t$. Substitute these random draws into Equations 3 and 4. At each time step sort the draws of the test statistic $\tilde{L}_t$ and extract the values at the 10, 50 and 100 row and check their sign versus the original L_t[3]. Finally, an average of the 10, 5 and 1% significance level and plot these lines versus the evolution of L_t. Three choices of loss function are used, which are labelled as the *matrix norm loss function*, the *inverse trace loss function* and the *maximum divergence loss function.* [4].

Matrix Norm Loss Function, (ML)

The matrix norm loss function is $f(\Sigma_1, \Sigma_2) = \|\Sigma_1 - \Sigma_2\|_F \equiv \sqrt{\sum_{i,j}^{n}(\sigma_{i,j,1} - \sigma_{i,j,2})^2}$, which is the Frobenius norm i.e. the square root of the sum of the squared differences between the realized covariance matrix $\Sigma_1 = \tilde{\Sigma}_t$ and the competing forecast matrices $\Sigma_2 = \{\Sigma^g_t, \Sigma^h_t\}$, the comparative loss function is therefore

$$\ell_t = \left\| \tilde{\Sigma}_t - \Sigma^g_t \right\|_F - \left\| \tilde{\Sigma}_t - \Sigma^h_t \right\|_F \tag{5}$$

This is therefore the comparative sum of absolute errors, between the competing forecasts.

Inverse Trace Loss Function, (IT)

The inverse trace loss function, $f(\Sigma_1, \Sigma_2) = tr(\Sigma_1 \Sigma_2^{-1} - I)^2$ is the trace of the product of the realized covariance matrix with the inverse of the forecast minus equivalently sized identity matrix I,

$$\ell_t = (tr\left(\Sigma^g_t \hat{\Sigma}^{-1}_t - I\right))^2 - (tr\left(\Sigma^h_t \hat{\Sigma}^{-1}_t - I\right))^2 \tag{6}$$

where tr is the trace of a matrix.

Maximum Divergence Loss Function, (MD)

The final loss function postulates the maximum feasible divergence between the realised and forecasted covariance matrix for a pair of n vectors $a \neq b \neq 0$, the generating loss function is, $f(\Sigma_1, \Sigma_2) = (\max_{a,b}(a\Sigma_1 b(a\Sigma_2 b)^{-1}) - 1)^2$, hence the comparative loss function,

[3]For the long run versus short run stability tests this is actually quite quick as all of the Cholesky factorisations are computed during the first pass when the covariance estimates are computed and the same matrices are used in each time step.

[4]Appendix B in the Internet Appendix discusses some aspects of the relative properties of the three loss functions. The matrix loss functions chosen here are the simplest parameter - free loss functions available and are multivariate extensions of those reviewed in Patton (2006)

$$\ell_t = \left(\max_{a,b\neq 0} a'\tilde{\Sigma}_t b(a'\Sigma_t^g b)^{-1} - 1\right)^2 - \left(\max_{a,b\neq 0} a'\tilde{\Sigma}_t b\big(a'\Sigma_t^h b\big)^{-1} - 1\right)^2 \quad (7)$$

The maximisation problem is solved using a standard quadratic programming procedure. For each loss function set Σ_t^h to be the long run CGP, with $p = 252$ days and Σ_t^g to be the short run CGP. Therefore positive values indicate that the long run forecast is preferred and negative values suggest that lower losses are associated with the short run forecasts. The normalised matrix loss functions are denoted by $L_t^{i\in\{ML,IT,MD\}}$ and the simulations are for the selected LSE stocks. The system is demonstrated for daily variations, therefore $m = 510$ minutes. To illustrate how the confidence intervals are built, the ML is implemented between the two potential CGPs. The distribution of the test statistics for a random draw from an $n-variate$ Wishart distribution with an identity generating matrix and n degrees of freedom is also demonstrated, in each case when neither candidate CGP is correct and when each one is correct.

In Figure 1 shows how the distribution of the average correlation changes as the system evolves over 1,000 iterations (the first 1,000 days are plotted to assist the reader, the simulations run over the same number of days as there are trading days in the sample). The top plot shows the situation when the system is long run stable. The centre line is the mid projection of the average correlation and the lines emanating outwards parallel to the centre line are in half standard deviation lots, running to plus and minus three standard deviations. For the long run stable CGP the mean correlation is expected to be constant, however within three standard deviations the correlation can substantially vary between 0.38 and 0.62. The second plot, is for the short run stable CGP, the projected mean correlation is slightly below 0.5, due to the truncation at one and minus one. The variation in average correlation is, however, far more pronounced. Within 100 iterations the three standard deviation range of the daily average correlation could be between minus one and plus one. At around 500 iterations the two standard deviation range covers minus one to one. Once the loss and corresponding test statistics are computed, then average across the random starting points and the pathways to compute the normalised confidence intervals.

[FIGURE 1 HERE]

4. Dataset and Pre Sampling

The underlying data set consists of all informative traded prices for 27 selected stocks from the NYSE, 25 from the LSE, 22 from the TSE and 20 from the FSE from January 1, 1996 to June 1, 2010[5]. The Thompson Reuters Tick History service records time and sales data for most available global equities. We restrict our sample to only those stocks that have been continuously traded over the whole sample period and only the largest traded companies within the selected markets. Stocks are ranked by cumulative trading volumes by year and we eliminate stocks that have more than two years of thin trading. The descriptive statistics for the sample are reported in Table 1.[6]

[TABLE 1 HERE]

For the US and Japanese stocks in the sample the Thomson-Reuters Tick History data are clean and few prices are adjusted or removed. However, for Germany and the UK data, prior to 2002, there are great number of rogue reported transactions. But most of these are $> 500\%$ higher than the local median price, therefore, are relatively easy to deal with.

However, the LSE and FSE data are slightly exceptional in this context. For instance, stocks from some exchange such as the Shanghai stock exchange, exhibited only 12 rogue trades for 86 cross listed stocks, albeit with a lower number of transactions per stock. Vodafone, in contrast, exhibited more 'rogue' transactions

[5] In this paper transaction data is used in the sample, future work intends to use both trade and quotes data.

[6] An issue occurred with the German stocks, as the data had a consistent gap from January 1, 1998 to January 1, 2002. The author would like to thank the data vendor for providing the missing tick information.

from the LSE or FSE in the period of January 1, 1996 to December 31, 2002 than all of the stocks from the Tokyo Stock Exchange (TSE) together. Therefore, days of data of stocks in these two exchanges are dealt with exclusion from the sample set where the total number of rogue transactions exceeds 1% of the total transactions for all stocks.

Ticks are pre-filtered by using the procedure suggested in Oomen (2005) to clean the data-set. This includes smoothing of instantaneous price inversions and the extraction and smoothing of business time returns greater than 8 standard deviations from a 20 minute moving average. The irregularly timed ticks are mapped to a standardised minute grid using a combination of moving average, importance sampling and, finally, a piecewise cubic spline. This yields $m = 390$ sampled observations per day for the New York traded stocks, $m = 510$ for London, $m = 270$ for Tokyo and $m = 480$ for Frankfurt. For each stock index a standardised business time is created for each available working day (holidays are recorded and an index is used to eliminate them from the sample).

Barndorff-Nielsen et al. (2009a, 2008) discuss the issue that how correctly mapping informative price ticks converted into returns can correctly assign directional shifts to changes in the fundamental price and microstructure noise. Audrino and Corsi (2008) address the off-diagonal issue by suggesting a return covariance matching, the so called "needlework" approach, to interpolating the returns and hence improving the accuracy of the covariance estimator. This approach has many benefits, however it is computationally intractable for large cross sections of data. To construct a high frequency covariance estimator the following approach is used: generate daily prices and tick-times of stocks at a high frequency sampling interval, in this case, second by second, including the opening or closing call auction if it is present. A grid of standardised tick-times is constructed and mapped onto the ticker history. A 12 second lead-lag moving average, incorporating lagged information from the last 20% of the preceding minutes/ticks and the following 20% of the next minute. The 12 second lead-lag moving average is then interpolated at the minute grid and this is the final recorded grid price. In practice, the second by second data is very irregularly spaced and depends on the underlying ticker updating. For certain stocks there are not enough updated ticks to populate the moving average and in this case a cubic spline across the tick grid is used to interpolate the missing price.

[FIGURE D.15 HERE] [A SINGLE PANEL OF TABLES D.2, D.3 AND 1]

Once the grid prices are interpolated returns are computed and subtracted from the daily average returns to form the daily excess returns matrix X_t. It may be considered a surprise, but there are many occasions where certain stocks have long periods before a new tick price occurs. If two or more stocks have long periods of delay in updating, then this results in a poorly conditioned sample covariance matrix. Moving to calendar time does not improve the situation as many stocks may have had updated ticks whilst the prices of a subset do not move at all. We overcome this issue by resampling with replacement across the cross section of stocks. The daily realised covariance matrix Σ is computed as well as the inverse conditioning number[7]. Inverse conditioning numbers near zero suggest a poorly conditioned matrix. A threshold of $10e - 4$ is set, below which a resampling algorithm with replacement is implemented. The rows of the excess return matrix, X_t, are resampled 100 times and the conditioning numbers of the resulting resampled covariance matrix are computed.

Figures D.17 and 2 plot the evolution of the equally average correlations and volatilities for the UK stocks at minute sampling frequency for different aggregations. A bootstrap resampling routine is used to construct the confidence intervals.

[A SINGLE PANEL HERE OF FIGURES D.17 AND 2]

5. Results and Analysis

In this section, we present our two main sets of results: those from the static matrix equality test and the results from the loss function approach across four markets. The equality tests demonstrate whether the

[7] The inverse conditioning number is the reciprocal of the ratio of p norms of the matrix and its inverse, $\left\|\hat{\Sigma}_t\right\|_{p=1} \cdot \left\|\hat{\Sigma}_t^{-1}\right\|_{p=1}$

short run and long run covariance matrix processes are equally mean reverting at different frequency across different markets. Therefore, we can test the stability of the actual dynamics of the covariance generating process (CGP).

[A SINGLE PANEL HERE OF FIGURES D.17 AND 2]

Tables D.5 to D.8 present the results of the covariance equality tests year by year (1996-2001) and for the whole sample period in the US, UK, Japanese and German stock markets. The test statistics of the equality tests are reported and we use ***, **, or * to indicate whether the statistics is significant at the 1, 5 and 10% levels. To assist readers, we follow the Muirhead (1983)procedure and divide the test and critical statistics by a constant(10e6). The results for the static matrix equality tests generally reject the null that the daily, monthly and quarterly covariance matrices are equivalent to the yearly and whole sample matrices for most of these markets at the 1% significance level. Only a few tests reject the null at the 5% or 10% significant levels; for instance, for the whole sample for the UK which fails to reject the null at 1%. This is very surprising given the expectation that the continuing market turmoil from the 2007-2010 financial crisis would create extra turbulence in the covariance structure. However, this only fails to reject at 1% significance but rejects at the 5% significant level.

The static test results are fairly surprising, given that even in years when there was a consistent bull market (2002-2006), the minute level covariance matrix varies enough that even for quarterly blocks the covariance matrix does not properly converge to the years covariance matrix, let alone the whole sample matrix. The results suggest that there is substantial variation in the covariance structure over the sample and at different scales and blocks of time. The non-rejection of the null for the whole sample for the German stocks may reflect the data problems mentioned in the 4 section when more extensive data cleaning required for this market.

[A SINGLE PANEL HERE OF TABLES D.5, D.6, D.7 AND D.8]

The uniformity of the results as well as the magnitude of the rejection is striking. This finding has a substantial implication for asset pricing models assuming long run covariance stability, in so far as the level of risk being exposed to is substantially mis-specified, when looking at the asset price dynamics at the minute level. A substantive point should be made here, sampling returns at a lower frequency (e.g. days, weeks or months) only smooths the covariance dynamics to a certain extent. Williams and Ioannidis (2010) demonstrate that if the underlying continuous time process exhibits stochastic covariance, then low frequency sampling will only hide the variation in covariance for a certain period (depending on the degree of variation in the CGP), however, eventually this property will emerge if the CGP is not mean reverting. The static results suggest that blocking the data in daily, monthly and quarterly blocks does not converge to the yearly or whole sample covariance matrices. This implies that the CGP is probably not mean reverting at a speed quicker than one quarter (or we would fail to reject equality for the quarterly blocks). Therefore, short sample back testing of correlation/covariance dynamics (e.g. using weekly data for say three years) could result in a catastrophic mis-specification of the level of risk, for covariance forecasts over a horizon of more than one week.

The equality tests do not identify the CGP but simply confirm that the covariance matrices for the chosen periods are not equal. With these results in hand, we can move on and compare how well the long and short run CGPs forecast the forward looking realised covariance over a variety of blocks of time.

The loss function component results aim to provide a robustness check of the stability of the covariance matrix by comparing different loss functions in generating covariances, namely: ML, IT and MD. Again, it is applied to daily, weekly and monthly levels across the four markets we study in this paper.

We report the evolution of the daily loss function test statistics with 90, 95 and 99% confidence intervals. The rolling windows are computed from 252 trading days, the slight kinks in the confidence intervals are caused by Monte-Carlo simulation adjustments. Only the results of daily loss functions are shown here in figures 3to 6. The rest loss functions results for the weekly, monthly and quarterly test statistics for these four markets provide similar conclusions to their daily findings and therefore, are not reported but available on request.

If the test statistic is in the negative domain, the short run CGP is outperforming the long run CGP. By contrast, if the test statistic is in the positive domain, then the long run CGP is outperforming the short run CGP. What is clear from these plots is that for long periods neither the ex-post covariance matrix or the ex-ante covariance matrix are completely adequate in describing the true CGP and this is supportive of the strong rejections from the static equality tests. Another striking aspect is that the three different loss functions (ML, IT and MD)produce very similar results. This is important for large dimensional systems as the ML loss function is computationally by far the simplest loss function to compute. This finding suggests that the degree of variation of the conditional covariance matrix is of a form that is consistent across loss functions and as such analysis of this kind may be robust to their particular choice.[8]

[FIGURE 3 HERE AS SINGLE PANEL WITH FIGURES 4, 5 AND 6 HERE]

The ML, IT and MD based test statistics are highly variable during the 2004-2010 period. All three loss functions produce comparative test statistics that suggest that neither loss function is adequate in describing the daily realized covariance matrix for most of the 1996-2003 period. However, the short run statistic is significantly preferred for a substantial epoch from 2004 to 2008. The pattern of the loss functions, when normalised, is virtually identical, with the ML loss function illustrating some slight increase in variability relative to the other two. The results are qualitatively robust to changes in the block size from weekly to quarterly blocks. The the pattern is slightly smoother (as expected), however, the trends are very similar to the daily blocks.

The CGPs for the LSE components offer very little predictive power over the whole sample, with only a short period in 2009, for the ML loss function generated test statistic, significantly favouring the short run CGP. Figures D.17 and 2 present the average correlation and standard deviations for the chosen LSE components for each day, week, month and quarter over the sample. The degree of variation is quite striking, with the crisis post 2007 represented by an extremely high spike in volatility and a substantial change in the correlation dynamics post 2007. The loss function results suggest that neither CGP appropriately captures the day to day, week on week or month on month shifts.

Similarly to the selected LSE components the dynamics of the covariance structure of the selected TSE components is not well captured by either the short or long run CGP. Matrix equality tests, see Table D.7, suggest that the long run CGP does not adequately describe the daily, weekly or monthly realized covariances. Rolling IT and MD loss function test statistics indicate that the short run CGP does significantly improve on the long run CGP over the 1996-2010 sample. Rolling ML loss function analysis suggests that the short run CGP significantly outperforms the long run CGP for brief periods, evenly scattered over the sample period.

In contrast to other markets, the daily realized covariance matrices of the FSE selected components exhibit more persistent bias towards one model over the other for all three loss functions. The rolling ML loss function test statistics shows a general tendency to favour the long run CGP with brief periods where the short run CGP is strongly favoured. However, matrix equality tests reject the long run CGP for daily, weekly, monthly and quarterly concatenations and this is implied by the marginal significance of the test statistic in the negative domain.

6. Conclusions and Future Directions

In this paper, a set of covariance stability tests have been outlined and implemented for selected stocks. In each case the CGP for each day, week, month and quarter are treated as a draw from a Wishart distribution. For each forecast the goodness of fit is compared a likelihood ratio test and using a matrix loss function approach. This method follows in the footsteps of the well established Diebold and Mariano (1995) framework for univariate forecast comparisons.

[8]The loss functions results for these four markets at the weekly, monthly and quarterly are mentioned in the discussions of the results in the following four subsections. The actual figures of such loss functions are available in the internet appendix.

These tests are then implemented using a high frequency data set for a large cross section of selected constituents sampled from four equity markets. The results indicate that neither of the two candidate CGPs, short run ex-ante and long run ex-post, adequately capture the true covariance dynamics. This is a very significant result as it implies that true CGP is far more variable than a simple Wishart autoregression (which the short run CGP actually is). The implications for portfolio management, risk measurement and many asset pricing models are stark, as most of these models rely on the assumption that correlations between assets and between assets and non-traded factors is, in the long run, mean reverting. This assumption includes popular models such as the ARCH/GARCH model and most standard stochastic volatility models. The fact that the covariance and correlation structure is not only stochastic, but is also not mean reverting suggests that most models of asset prices are only valid for particular epochs and that in the long run no model with mean reverting second moments can fully capture the overall risk exposure.

In two recent contributions Williams and Ioannidis (2011) and Buraschi et al. (2010) have suggested methods to construct and price instruments that hedge correlation and covariance risk. The results of this study suggest that without such instruments proper control of the risk of equity portfolios is not possible. However, given the difficulty in identifying the correct CGP the parameters of the pricing kernel for such instruments would be difficult to identify.

Future research in this area should attempt to isolate better performing CGPs than the simple short and long run types used in this study. Alternative CGPs, such as the Wishart stochastic covariance models, which might be of use in certain contexts are presented in the Internet Appendix.

[TABLE D.4 HERE]

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7. FIGURES AND TABLES

Table 1: Descriptive Statistics

The first section of the table reports the minute level descriptive statistics. The second section reports the number of included stocks, total exchange closure days and the total size of the minute grid. The third section reports what proportion of the grid data is actually a real observation (i.e. the actual price within 100 milliseconds) or an interpolated price using the cubic spline procedure outlined in §(4). *This includes the 'Flash Crash' of May 6, 2010.

	US*	UK	Japan	Germany
Mean	0.00039100	-0.00018000	0.00932700	-0.00022400
Standard Deviation	0.02295000	0.03019000	0.02084860	0.02057100
Median	-0.00000000	-0.00000000	0.00000100	-0.00002900
Minimum	-0.49741000	-1.88744200	-1.83025300	-0.36240000
Maximum	0.47130000	1.92619300	0.82711800	0.36561000
Skewness	0.07568100	1.28767200	-37.73241100	0.06990200
Kurtosis	130.15257000	8605.69242300	6951.26492700	132.90961000
No. Stocks	27	25	22	20
Grid Mins.	390	510	270	480
Candidate Grid Days	3,766	3,766	3,766	3,766
Holidays and Excluded	111	119	136	222
Total Minute Obs	1,415,700	1,859,970	956,880	1,754,400
Usable Obs.	31,519,123	39,628,942	13,787,592	24,645,142
Grid Obs.	38,223,900	46,499,250	21,051,360	35,088,000
Adjusted Obs.	6,704,777	6,870,308	7,263,768	10,442,858

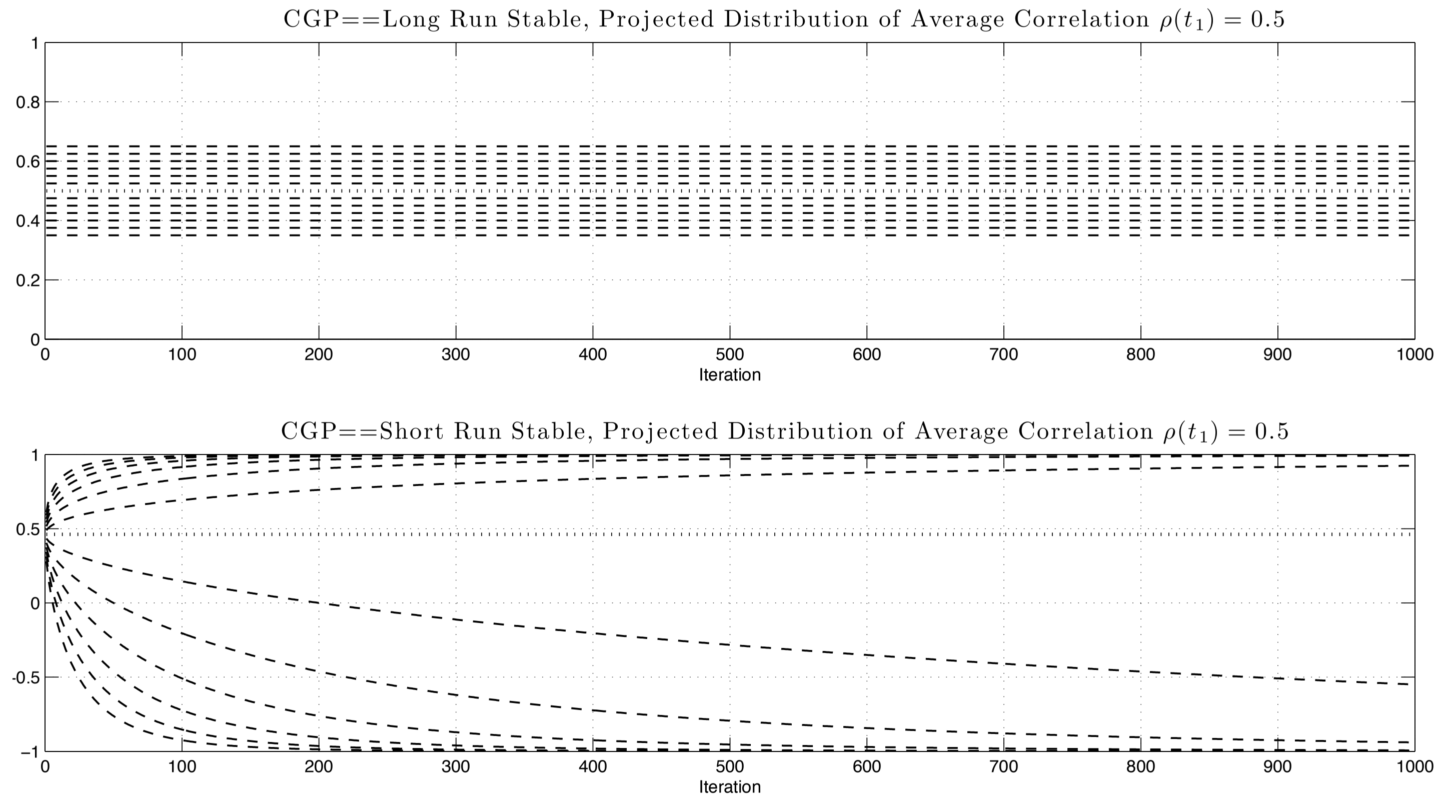


Figure 1: The projected variation in average correlation for 24 simulated assets over 1,000 pathways, with an initiating average correlation of 50%. The dotted lines present the central expected value at t_1, the dashed lines represent the variation in average correlation between 0.5 and 3 standard deviations

Figure 2: Example: The Volatility Dynamics for the UK Market.

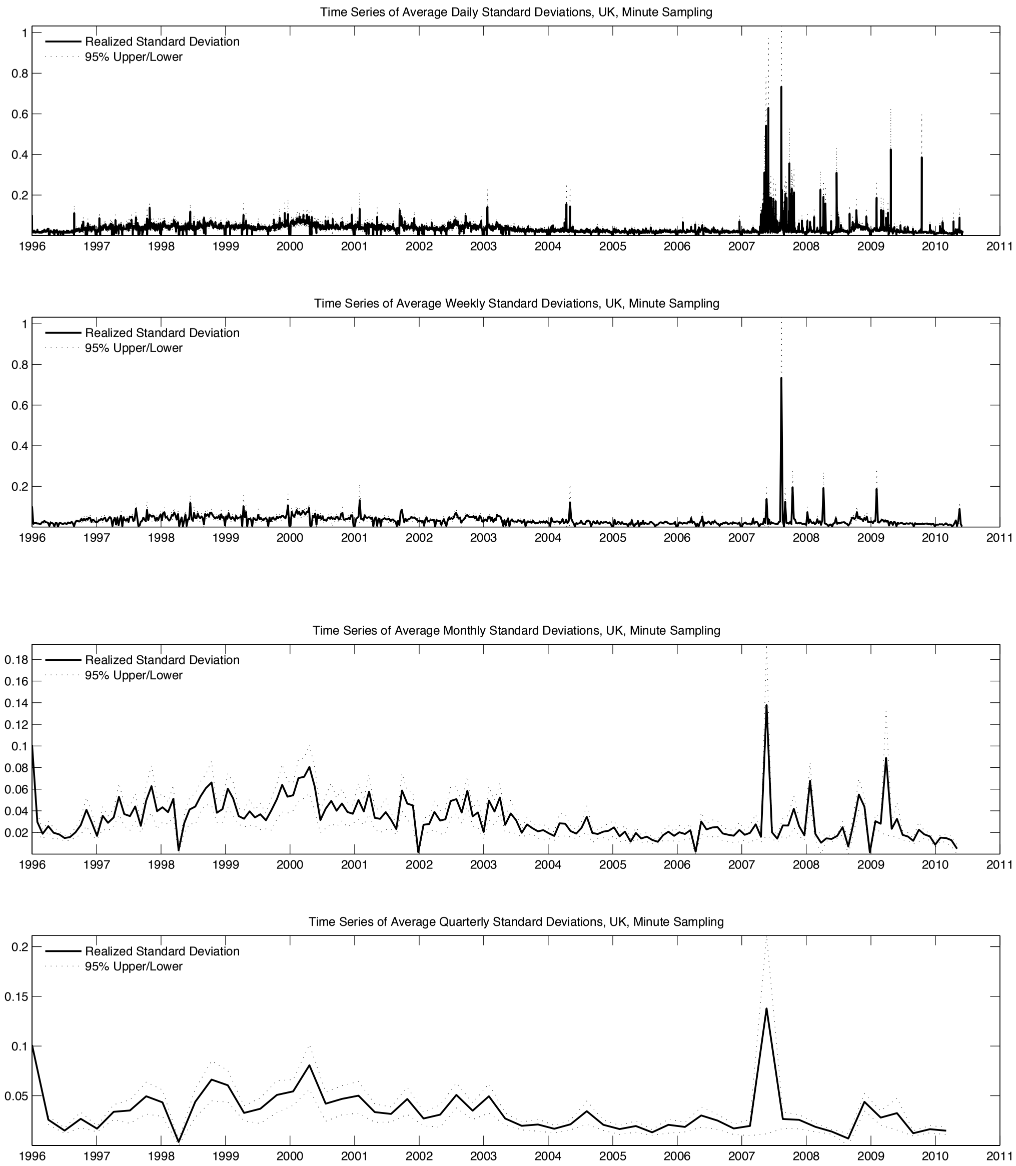


Figure 3: Loss Function Plots and Significance Bounds for Selected NYSE Components

The evolution of the test statistic for the three loss functions types for the selected constituent stocks from the NYSE, LSE, TSE and FSE. If the test statistic is above the zero axis then the short run test covariance forecast, Σ_t^h, is favoured. If it is below the zero axis then the long run forecast, Σ_t^g, is favoured. Upper and lower critical bounds (ordered away from the zeros axis at 90, 95 and 99% respectively are presented as the black dashed lines. The rolling window looks back 252 days (average 1 trading year and is adjusted for holidays).

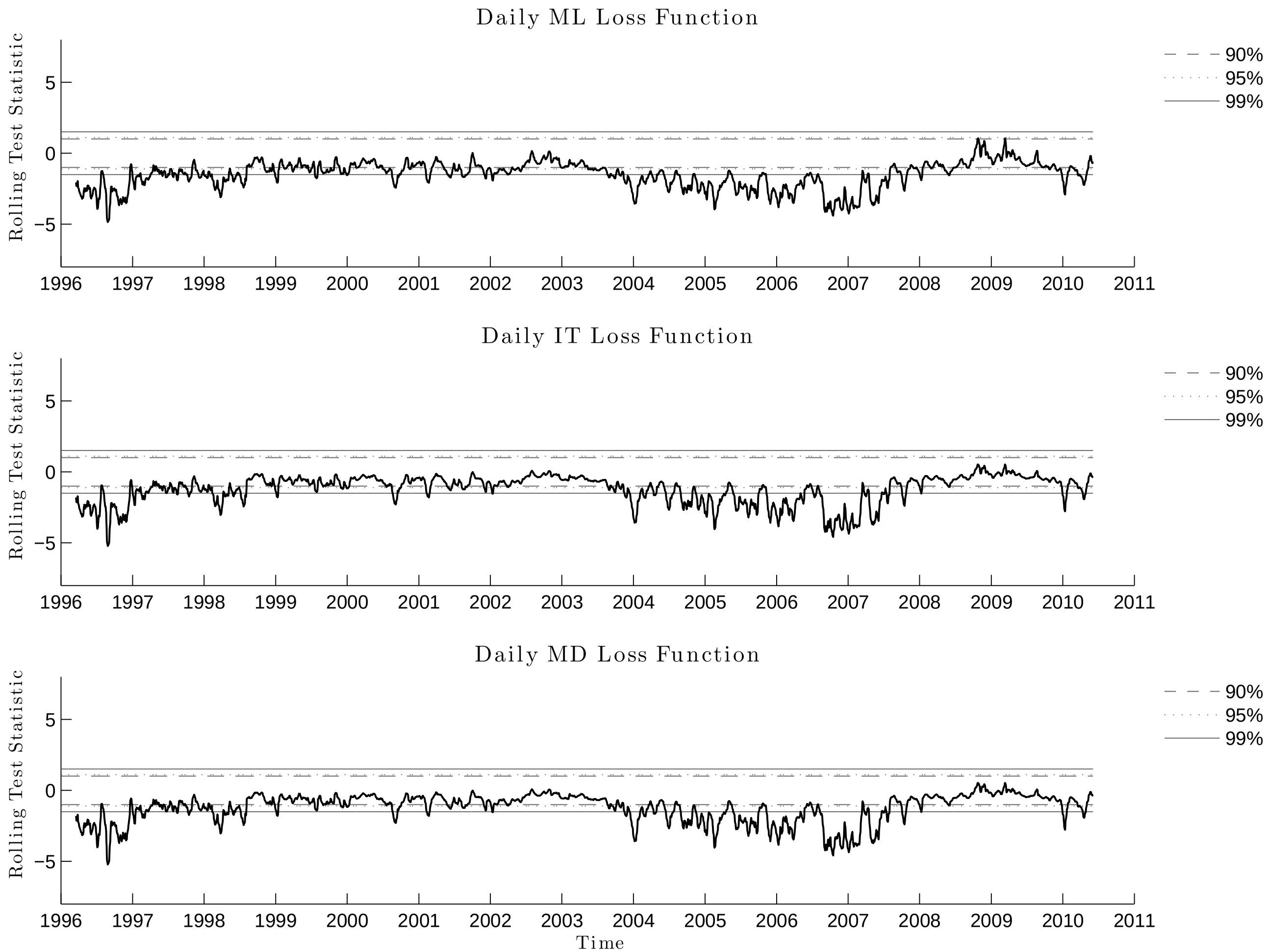


Figure 4: Loss Function Plots and Significance Bounds for Selected LSE Components

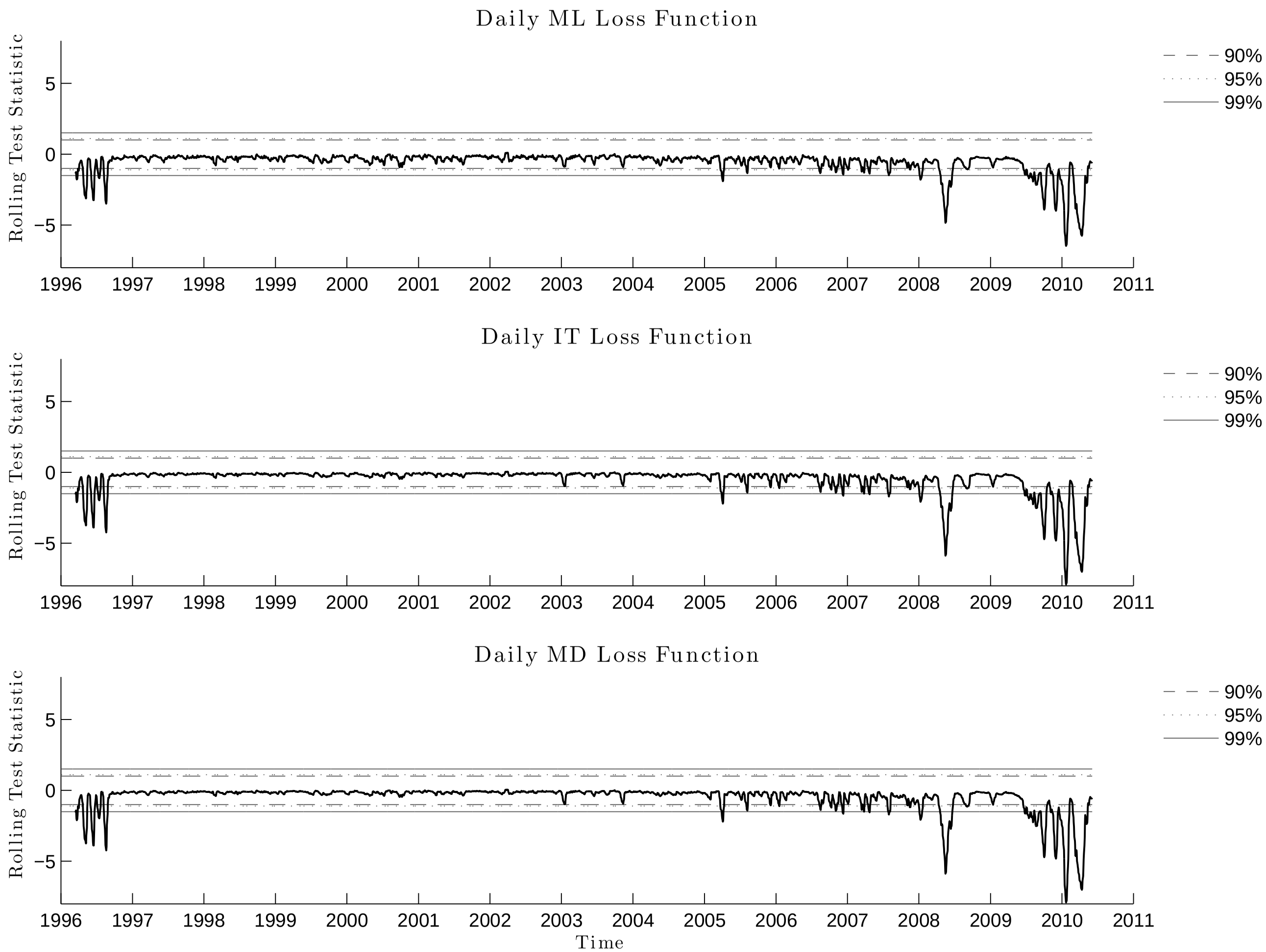


Figure 5: Loss Function Plots and Significance Bounds for Selected Tokyo Stock Exchange Components

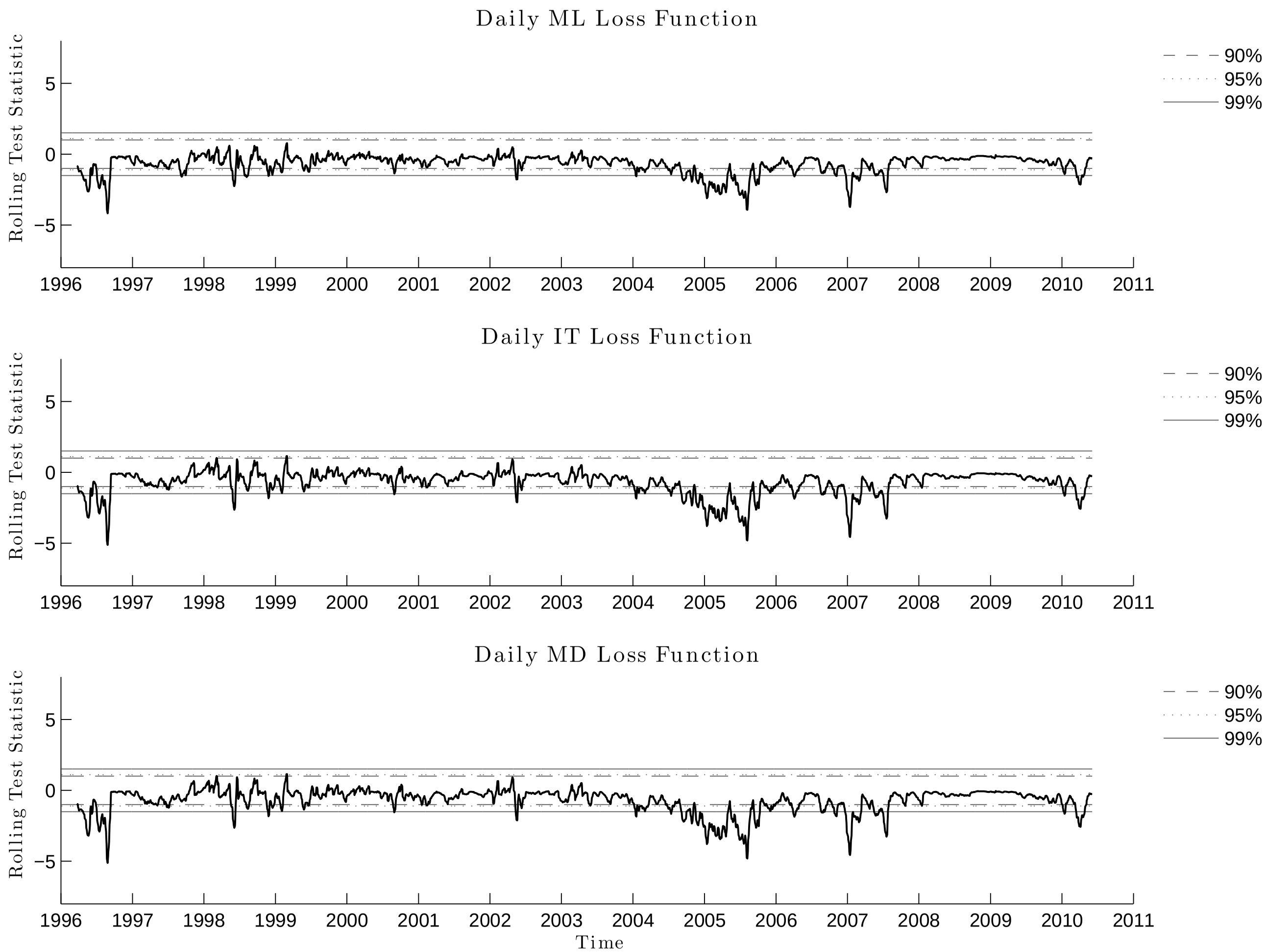
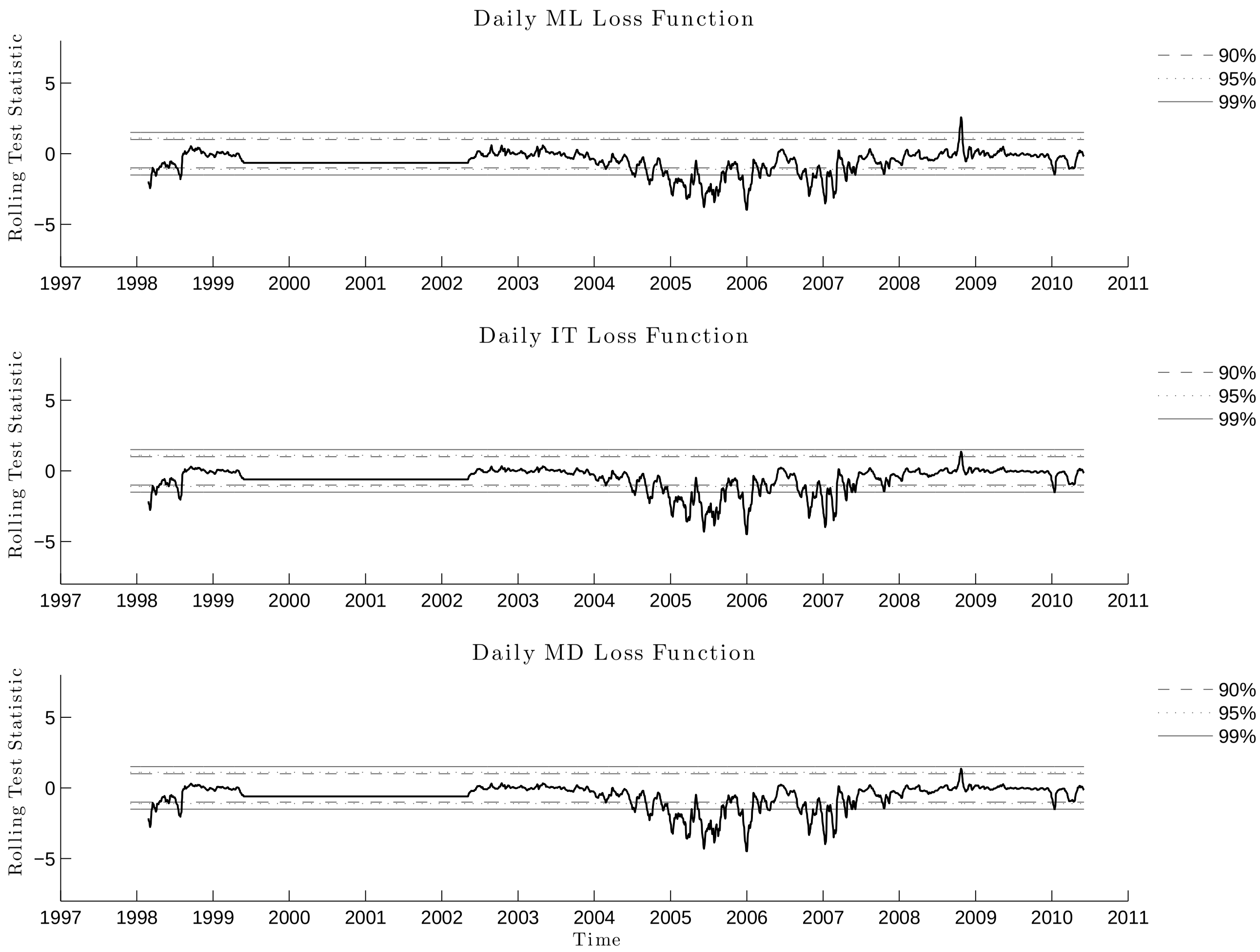


Figure 6: Loss Function Plots and Significance Bounds for Selected Frankfurt Stock Exchange Components



INTERNET APPENDIX

Appendix A. Further Details of Data Cleaning

This appendix provides further details of data sorting and grid mapping in order to get the final clean data set for the modeling.

Further cleaning of the data is more complex and several prior studies have provided discussion on how to pre-filter the data without 'over cleaning' it, see Barndorff-Nielsen et al. (2009b). Most of the filtration is on the price level and not returns as this tends to be easier to verify and less likely to result in systematic contamination of the return series that must then be corrected in the covariance computation stage. Cross market analysis such as this one has the added complication that the source of contaminants in the data is systematically different for different exchanges. In addition, the level of contamination varies through time.

Details of further filtration of these transactions is as follows: first, the day trades are partitioned into a morning and afternoon session. For the Tokyo case we use the exchanges break at 11am. and prices are sorted for the two periods and used to compute the median price for each $\bar{x}_{i,t}$. The log difference is computed for each price relative to the median price. Prices that are more than 300% different to the median price, which is identified as being a rogue transaction, are identified and rescaled relative to their neighbouring prices. Excluding these prices completely, however risks eliminating potentially important information that these rogue transactions may have.

Figure D.16 illustrates how the data and standardised time stamps are arranged for each day. The Softbank Corp. (RIC Code 9984.T) is shown as an illustration of how the day, week, month and quarter blocks of data are constructed. At the minute frequency the main data matrix is very large, for example the UK data matrix is 1,920,660 by 24 and is 352 Mb in size. Manipulation of this matrix can lead to memory issues, however a 64 bit operating system can accommodate very large memory blocks up to 8Tb and as such the all of the data and relevant projections can be loaded into memory.

Appendix B. Properties of Loss Functions

The ML and IT loss functions have analytic structures and the properties of these two loss functions are easily explored. The MD loss function is computed numerically by quadratic programming, the properties of this loss function are explored using a numerical simulation approach.

Consider the (at least) twice differentiable function $f : (\mathbb{C}^{n\times n}, \mathbb{C}^{n\times n}) \rightarrow \mathbb{R}_+$, for any two matrices $\{\Sigma_1, \Sigma_2\} \in \mathbb{C}^{n\times n}$, where $[\sigma_{i,j,k}]$ is the $\{i,j\}$ element of matrix $k \in \{1,2\}$ setting $\lambda = f(\Sigma_1, \Sigma_2)$, the gradient h and hessian H are defined as follows,

$$\nabla f(\Sigma_1, \Sigma_2) = \left[\frac{\partial\lambda}{\partial\sigma_{i,j,k}}\right] = h(\sigma_{i,j,k}) \tag{B.1}$$

$$\nabla^2 f(\Sigma_1, \Sigma_2) = \left[\frac{\partial^2\lambda}{\partial\sigma_{i,j,k}\partial\sigma_{i,j,k}}\right] = H(\sigma_{i,j,k}, \sigma_{i,j,k}) \tag{B.2}$$

Partitioning the gradient,

$$h(\sigma_{i,j,k}) = [h(\sigma_{i,j,1}), h(\sigma_{i,j,2})]' \tag{B.3}$$

$$H(\sigma_{i,j,k}, \sigma_{i,j,k}) = \begin{bmatrix} H(\sigma_{i,j,1}, \sigma_{i,j,1}) & H(\sigma_{i,j,1}, \sigma_{i,j,2}) \\ H(\sigma_{i,j,1}, \sigma_{i,j,2}) & H(\sigma_{i,j,2}, \sigma_{i,j,2}) \end{bmatrix} \tag{B.4}$$

Definition 1: Convergence

A function $f(\Sigma_1, \Sigma_2)$ is a convergent loss function if for when $\lambda = 0$, $\Sigma_1 = \Sigma_2$ and for when $\lambda > 0$, $\Sigma_1 \neq \Sigma_2$.

Definition 2: Equivalence

A twice differentiable loss function is element by element equivalent if and only if for $\frac{\partial\lambda}{\partial\sigma_{i,j,k}} = \delta$ and $\frac{\partial^2\lambda}{\partial\sigma_{i,j,k}\partial\sigma_{i,j,k}} = \varepsilon, \quad \forall i,j,k$, where δ and ϵ are constants.

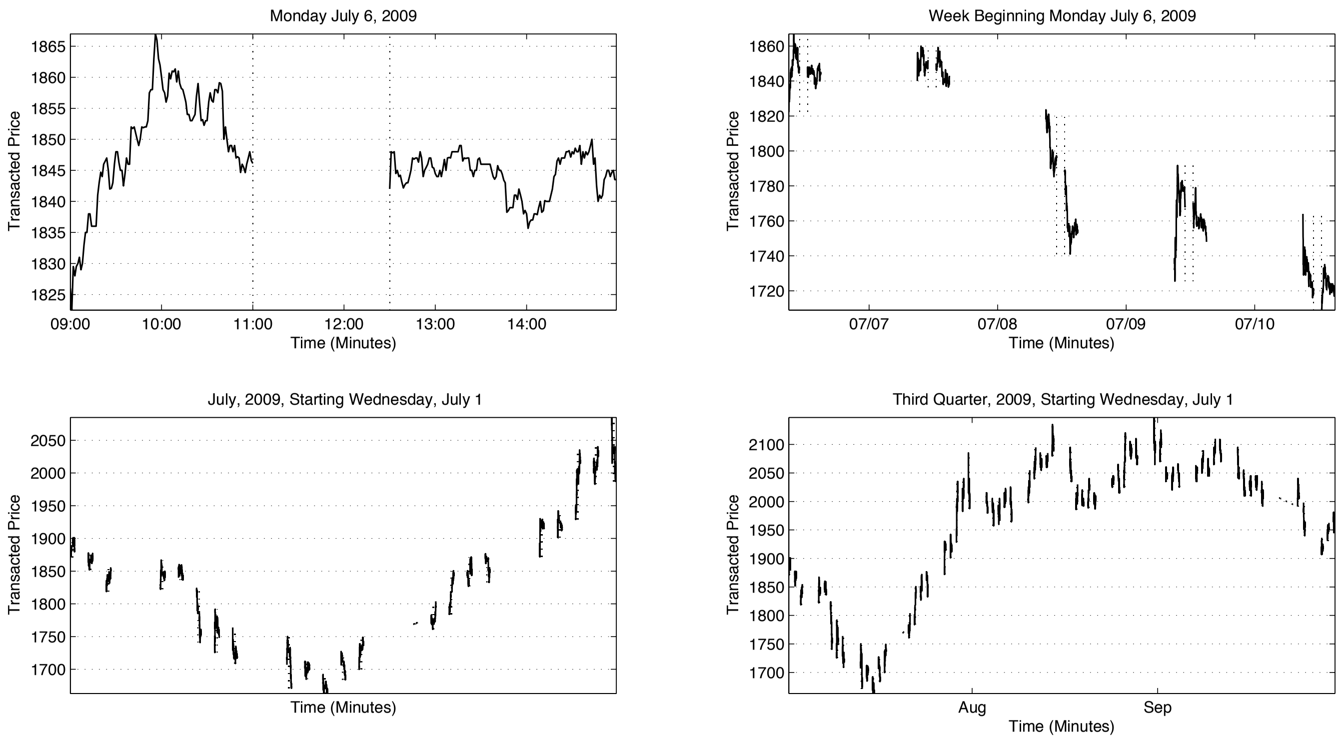


Figure A.7: Example of the data sorting and grid mapping. The data here is for Quarter 3 of 2009. This quarter begins Wednesday, July 1, 2009 and runs to Wednesday, July 30, 2009. Plot (1,1) illustrates the data for the first Monday of July, July 6, 2009. The vertical dotted lines denote the break period, when the Japanese market ceases to trade. All transaction prices in this window are ignored and the returns from the last and first minute either side of the break are also ignored. Plot (1,2) illustrates the first week of July, running from July 6, 2009 to July 10, 2009. Plot (2,1) illustrates the set of 22 days data for July 2009. On Monday, 20, July 2009 the stock is not traded (this is the 14th day of data on the plot), no data for this day is recorded and this day is excluded from the covariance test statistic calculation. If the day is a public holiday then the day is completely excluded, however some days a stock is either thinly traded or not traded at at all, however the market is open. These days are completely excluded from the computation of the covariance equality test statistic, however these days are recorded as a zero contribution to the rolling loss tests, by restricting the column and row of the realised covariance matrix and its forecasts to vectors of zeros. Plot (2,2) shows the entire quarter of data, 3 more non trading days are obvious in the 8th 7th and 6th from last days, again the data for these days is excluded. For each stock a list of non-traded days and other relevant corporate actions (such as stock splits and repurchases) are compiled.

Definition 3: Symmetry

A loss function is symmetrical if for a constant distance $\xi_{i,j} = \sigma_{i,j,1} - \sigma_{i,j,2}$, $\lambda(\xi_{i,j}) = \lambda(-\xi_{i,j})$, $\quad \forall i,j$.

Definition 4: Accelerative

A loss function is accelerative if $\frac{\partial^2 \lambda}{\partial \sigma_{i,j,k} \partial \sigma_{i,j,k}} = g(|\xi_{i,j}|)$, where $g(\cdot)$ is a strictly increasing function w.r.t. $\xi_{i,j}$.

Appendix B.1. Properties of Chosen Loss Functions

The various properties of the chosen loss functions are evaluated starting with the simplest, the ML loss function.

Proposition 1a: ML Convergence

The ML loss function $f(\Sigma_1, \Sigma_2) = \|\Sigma_1 - \Sigma_2\|_F$ is convergent.

Proof 1a: ML Convergence

Setting $\Xi = \Sigma_1 - \Sigma_2$, when $\Sigma_1 = \Sigma_2$, $\Xi = [0]$, where $[0]$ is an $n \times n$ matrix of zeros. $\|[0]\|_{p=1}^2 = \sum_{i,j} 0^2 = 0$. When $\Sigma_1 \neq \Sigma_2$, $\Xi \neq [0]$, $\|\Xi\|_{p=1}^2 = \sum_{i,j} \xi_{i,j}^2$ and subsequently by definition $\sum_{i,j} \xi_{i,j}^2 > 0 \quad \blacksquare$

Proposition 1b: ML Equivalence

The ML loss function $f(\Sigma_1, \Sigma_2) = \|\Sigma_1 - \Sigma_2\|_{p=2}$ is equivalent and symmetric.[9]

Proof 1b: ML Equivalence and Symmetry

For the ordered pair of elements $(\sigma_{i,j,1}, \sigma_{i,j,2})$,

$$\lambda(\xi_{i,j}) = \|\Xi\|_{p=2} \tag{B.5}$$

$$= \sqrt{\sum_{i,j} (\sigma_{i,j,1} - \sigma_{i,j,2})^2} \tag{B.6}$$

therefore by symmetric construction, $\frac{\partial \lambda(\xi_{i,j})}{\partial \sigma_{i,j,1}} = \frac{\partial \lambda(\xi_{i,j})}{\partial \sigma_{i,j,2}} = \delta$, similarly for the second derivative $\frac{\partial^2 \lambda}{\partial \sigma_{i,j,1} \partial \sigma_{i,j,1}} = \frac{\partial^2 \lambda}{\partial \sigma_{i,j,2} \partial \sigma_{i,j,2}}$ and $\frac{\partial^2 \lambda}{\partial \sigma_{i,j,1} \partial \sigma_{i,j,2}} = \frac{\partial^2 \lambda}{\partial \sigma_{i,j,2} \partial \sigma_{i,j,1}} = \frac{\partial^2 \lambda}{\partial \sigma_{i,j,1} \partial \sigma_{i,j,1}} = \varepsilon \quad \blacksquare$

Proof 1c: ML Acceleration

The ML loss function $f(\Sigma_1, \Sigma_2) = \|\Sigma_1 - \Sigma_2\|_{p=2}$ is not accelerative.

Proof 1c: ML Acceleration

For a given direction $\partial \sigma_{i,j,k}$, basic differentiation dictates that:

$$\partial \lambda(\xi_{i,j}) = \sqrt{\sum_{i,j} \left(\xi'_{i,j}\right)^2} - \sqrt{\sum_{i,j} (\xi_{i,j})^2} \tag{B.7}$$

$$= \sqrt{(\partial \sigma_{i,j,k})^2} \tag{B.8}$$

$$= |\partial \sigma_{i,j,k}| \tag{B.9}$$

Therefore by construction all $\frac{\partial \lambda(\xi_{i,j})}{\partial \sigma_{i,j,k}} = 1 \quad \blacksquare$

[9] The drawback here is that the loss function is element by element symmetric, however over the domain of available positive definite covariance matrices, this property is bounded, see the numerical examples below.

Proposition 2a: IT Convergence

The IT loss function $f(\Sigma_1, \Sigma_2) = \left(tr\left(\Sigma_1\Sigma_2^{-1} - I\right)\right)^2$ is convergent.

Proof 2a: IT Convergence

When $\Sigma_1 = \Sigma_2$, $\Sigma_1\Sigma_2^{-1} = I$, therefore $(I - I)^2 = 0$. When $\Sigma_1 \neq \Sigma_2$, the uniqueness of invertible matrices states that for an inverse $Q = \Sigma_1^{-1}$ of an invertible matrix Σ_1, Q is unique, therefore for another invertible matrix $\Sigma_2 \neq \Sigma_1$, with inverse P, $\Sigma_1 P \neq I$. As such, $tr(\Sigma_1 P - I) \neq 0$, therefore $\left(tr\left(\Sigma_1\Sigma_2^{-1} - I\right)\right)^2 > 0$ ■

Proposition 2b: IT Equivalence and Symmetry

The IT loss function $f(\Sigma_1, \Sigma_2) = \left(tr\left(\Sigma_1\Sigma_2^{-1} - I\right)\right)^2$ is not necessarily equivalent or symmetric.

Proof 2b: IT Equivalence and Symmetry

The proof is by use of constructing a counter example. If $f(\Sigma_1, \Sigma_2) = \left(tr\left(\Sigma_1\Sigma_2^{-1} - I\right)\right)^2$ is equivalent and symmetric, then $\frac{\partial\lambda}{\partial\sigma_{i,j,k}} = \delta, \quad \forall i, j, k$. Consider the 2×2 pair of invertible matrices

$$\Sigma_1 = \begin{bmatrix} \sigma_{1,1,1} & \sigma_{1,2,1} \\ \sigma_{1,2,1} & \sigma_{2,2,1} \end{bmatrix}, \Sigma_2 = \begin{bmatrix} \sigma_{1,1,2} & \sigma_{1,2,2} \\ \sigma_{1,2,2} & \sigma_{2,2,2} \end{bmatrix} \tag{B.10}$$

computing the functional form of the loss function is:

$$\lambda = \left(-2 - \frac{2\sigma_{1,2,1}\sigma_{1,2,2}}{-\sigma_{1,2,2}^2 + \sigma_{1,1,2}\sigma_{2,2,2}} + \frac{\sigma_{1,1,2}\sigma_{2,2,1}}{-\sigma_{1,2,2}^2 + \sigma_{1,1,2}\sigma_{2,2,2}} + \frac{\sigma_{1,1,1}\sigma_{2,2,2}}{-\sigma_{1,2,2}^2 + \sigma_{1,1,2}\sigma_{2,2,2}}\right)^2 \tag{B.11}$$

differentiation with respect to $\sigma_{1,1,1}$ and $\sigma_{1,1,2}$

$$\frac{\partial\lambda}{\partial\sigma_{1,1,1}} = \frac{-2\sigma_{2,2,2}(2\sigma_{1,2,1}\sigma_{1,2,2} - 2\sigma_{1,2,2}^2 - \sigma_{1,1,2}\sigma_{2,2,1} - \sigma_{1,1,1}\sigma_{2,2,2} + 2\sigma_{1,1,2}\sigma_{2,2,2})}{(\sigma_{1,2,2}^2 - \sigma_{1,1,2}\sigma_{2,2,2})^2} \tag{B.12}$$

$$\frac{\partial\lambda}{\partial\sigma_{1,1,2}} = \frac{2(-2\sigma_{1,2,1}\sigma_{1,2,2} + 2\sigma_{1,2,2}^2 + \sigma_{1,1,2}\sigma_{2,2,1} + \sigma_{1,1,1}\sigma_{2,2,2} - 2\sigma_{1,1,2}\sigma_{2,2,2})(\sigma_{1,2,2}^2\sigma_{2,2,1} - 2\sigma_{1,2,1}\sigma_{1,2,2}\sigma_{2,2,2} + \sigma_{1,1,1}\sigma_{2,2,2}^2)}{(\sigma_{1,2,2}^2 - \sigma_{1,1,2}\sigma_{2,2,2})^3} \tag{B.13}$$

given that clearly $\frac{\partial\lambda}{\partial\sigma_{1,1,1}}$ is not always equal to $\frac{\partial\lambda}{\partial\sigma_{1,1,2}}$ then the IT loss function is not equivalent (and by definition not symmetric)■

Proposition 2c: IT Acceleration

The IT loss function is not accelerating w.r.t $\sigma_{i,j,k}$.

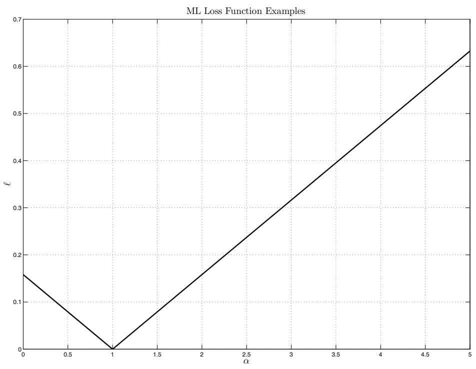
Proof 2c: IT Acceleration

Again, by counter example, differentiating B.12 again with respect to $\sigma_{1,1,1}$, yields $\frac{2\sigma_{2,2,2}^2}{(\sigma_{1,2,2}^2 - \sigma_{1,1,2}\sigma_{2,2,2})^2}$, which is not a continuously increasing function w.r.t. $\sigma_{1,1,1}$ ■

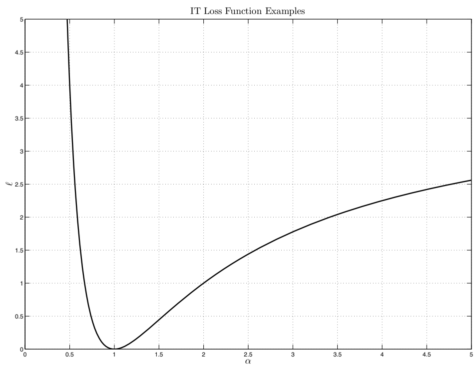
Properties of MD Loss Function

The MD loss function is based on the covariance equality test developed by Roy (1953). The loss profile associated with the function $f(\Sigma_1, \Sigma_2) = \left(\max_{a,b}\left(a\Sigma_1 b(a\Sigma_2 b)^{-1}\right) - 1\right)^2$ is computed by numerical optimisation. By construction when $\Sigma_1 = \Sigma_2$, $\lambda = 0$ as for any n length non-zero vectors a and b in the vector spaces $\mathcal{A}$ and $\mathcal{B}$, $a\Sigma_1 b = a\Sigma_2 b$ when $\Sigma_1 = \Sigma_2$.

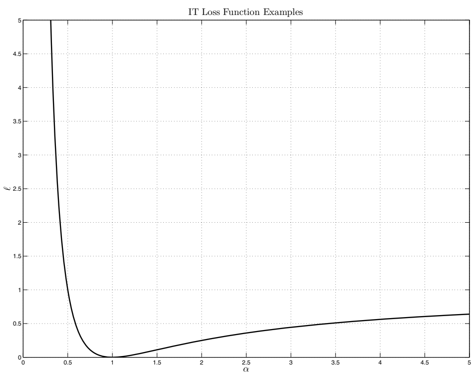
For demonstration purposes a 2×2 example is used. Setting $\Sigma_1 = \begin{bmatrix} 0.1 & 0.05 \\ 0.05 & 0.1 \end{bmatrix}$ and $\Sigma_2 = \alpha\Sigma_1, \quad 0.01 < \alpha < 5$, Figure 8(c) plots the loss function over the specified range of α, recall that when $\alpha = 1$, $\Sigma_1 = \Sigma_2$. The graphic illustrates the MD loss function relative to the ML and IT functions for this particular example.



(a) ML



(b) IT



(c) MD

Figure B.8: Evaluation of loss functions w.r.t. $\Sigma_2 = \alpha\Sigma_1, \quad 0.01 < \alpha < 5$. Optimisation for the MD loss function is carried out using the `fmincon` sequential quadratic programming minimisation algorithm in MatLab.

Appendix C. Loss Functions For Weekly, Monthly and Quarterly Projections

Appendix D. Other Figures and Tables

Comparative Loss Functions For Rolling Weekly, Monthly and Quarterly Projections For Selected NYSE Components.

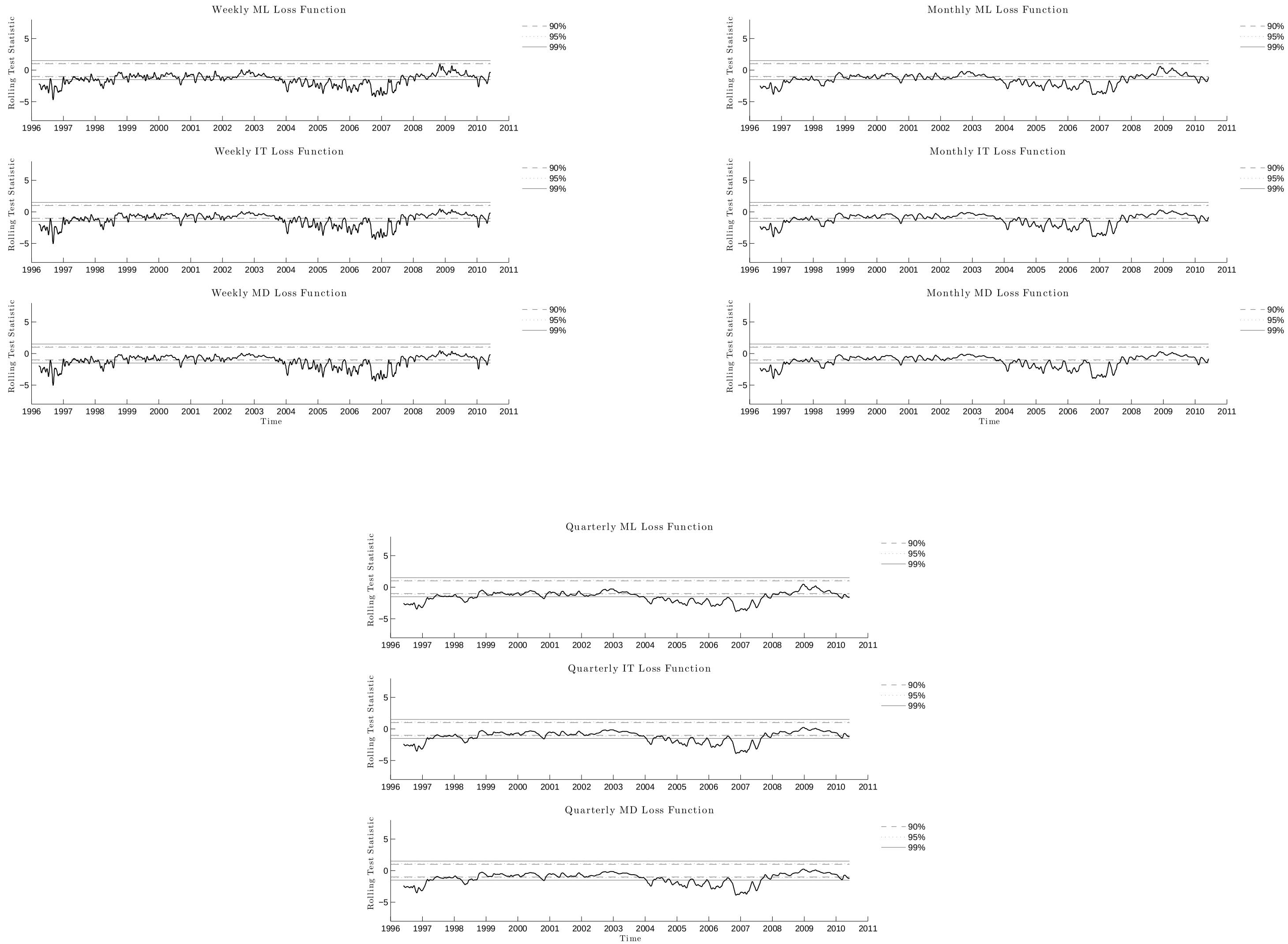


Figure C.9: Rolling Comparative Loss Statistics for US

Comparative Loss Functions For Rolling Weekly, Monthly and Quarterly Projections For Selected LSE Components

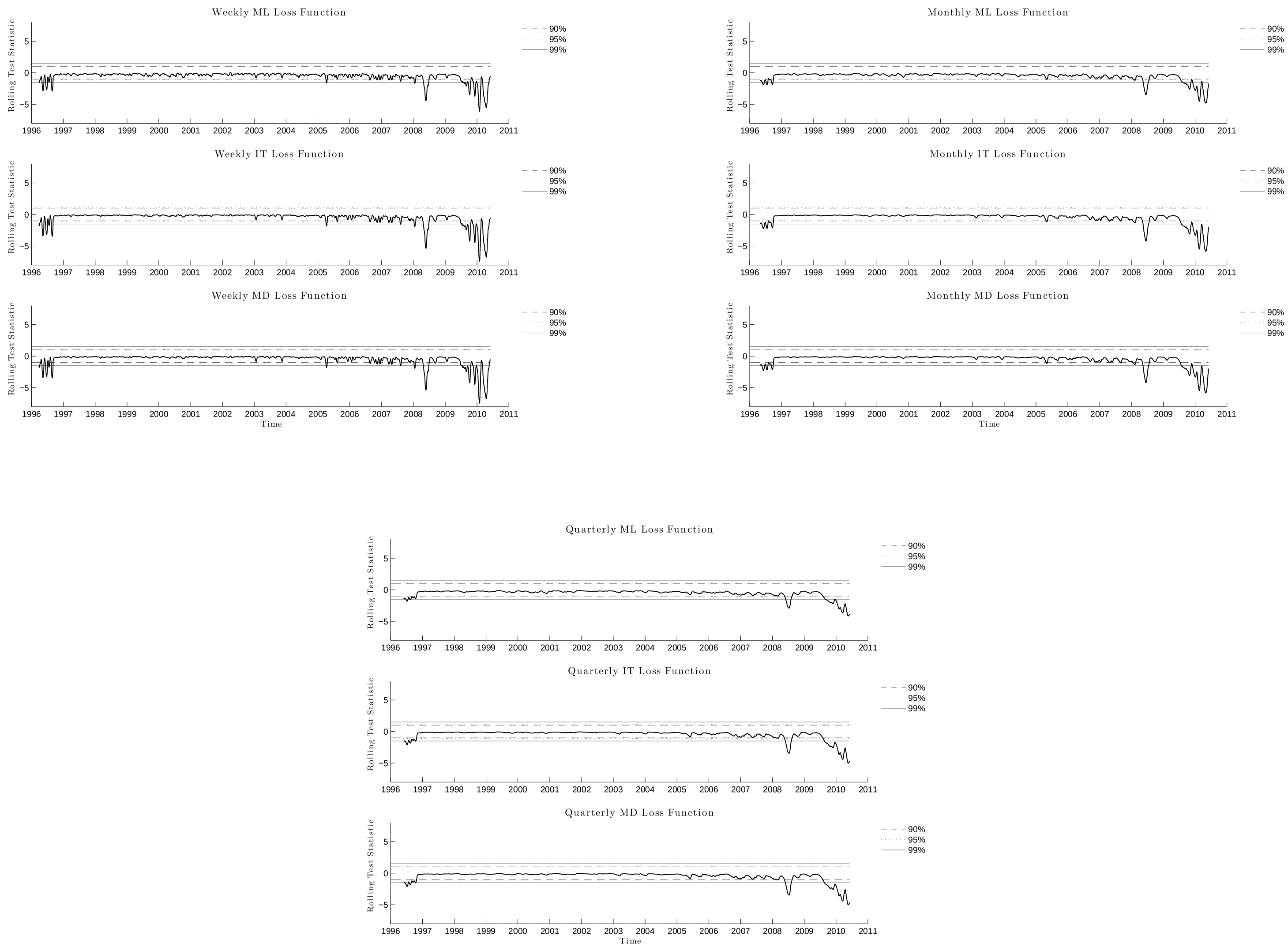


Figure C.10: Rolling Comparative Loss Statistics for UK

Comparative Loss Functions For Rolling Weekly, Monthly and Quarterly Projections For Selected TSE Components

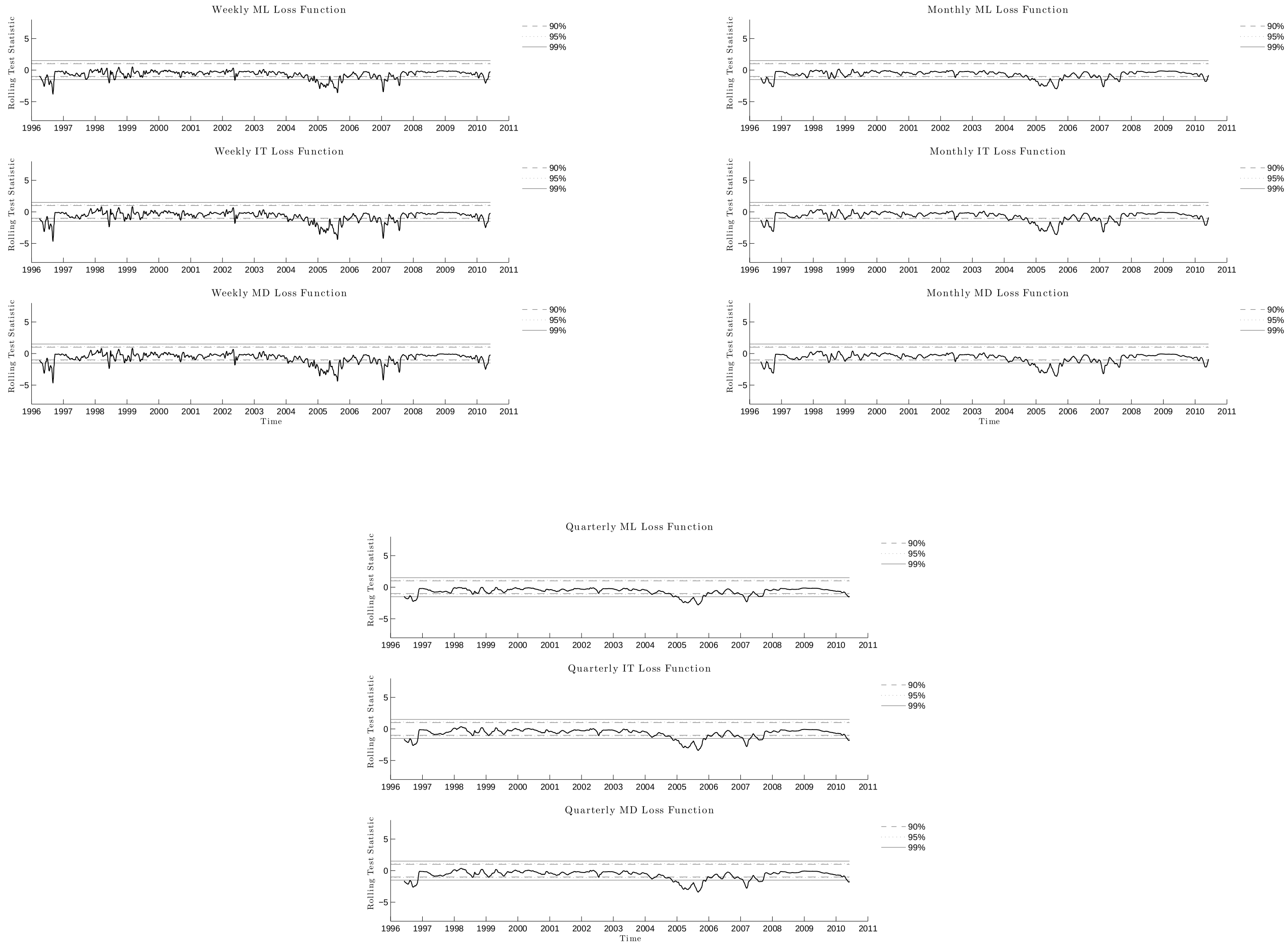
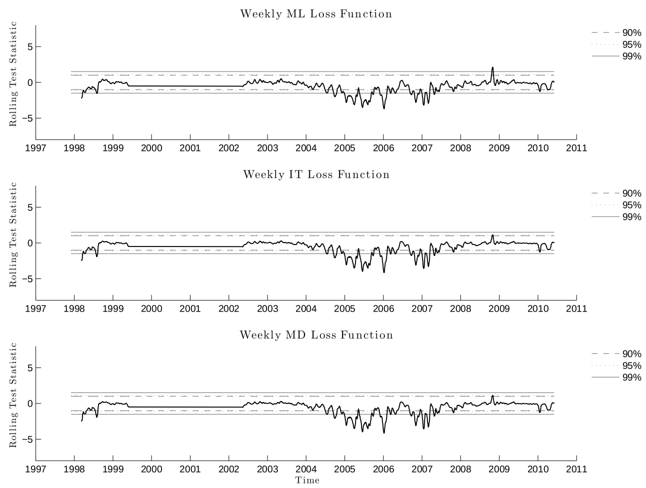
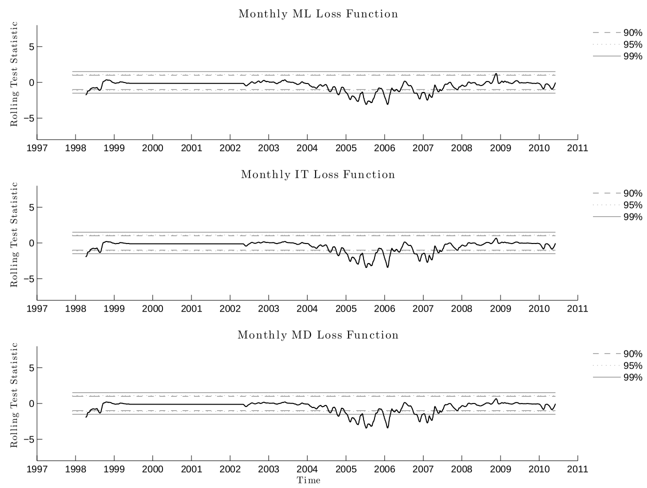


Figure C.11: Rolling Comparative Loss Statistics for Japan

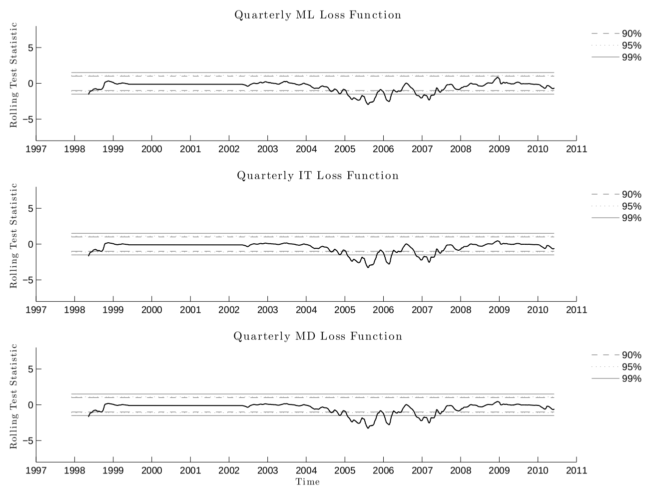
Comparative Loss Functions For Rolling Weekly, Monthly and Quarterly Projections For Selected FSE Components



(a) Weekly



(b) Monthly



(c) Quarterly

Figure C.12: Rolling Comparative Loss Statistics for Germany

Figure D.13: The CDFs of the test statistic under various CGPs. The top left plot presents the CDF of the test statistic when the CGP is short run stable. The top right presents the CDF of the test statistic when the CGP is long run stable. The dotted line straddling the two is when neither CGP is correct and the true CGP is a random draw from a Wishart matrix distribution with an identity generating matrix and 24 degrees of freedom.

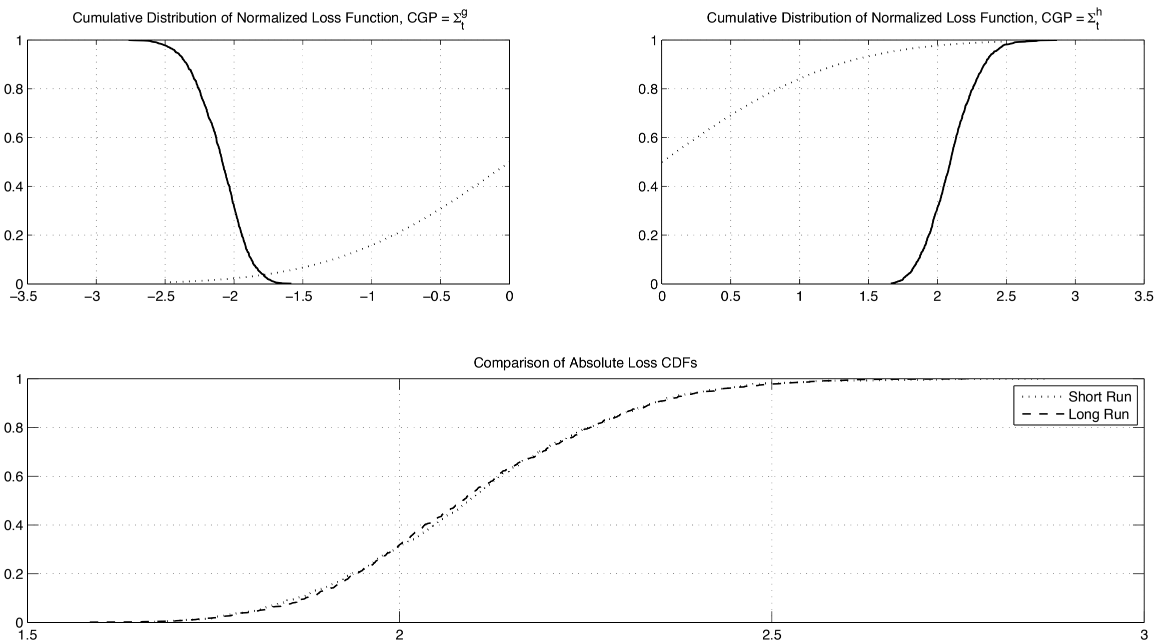
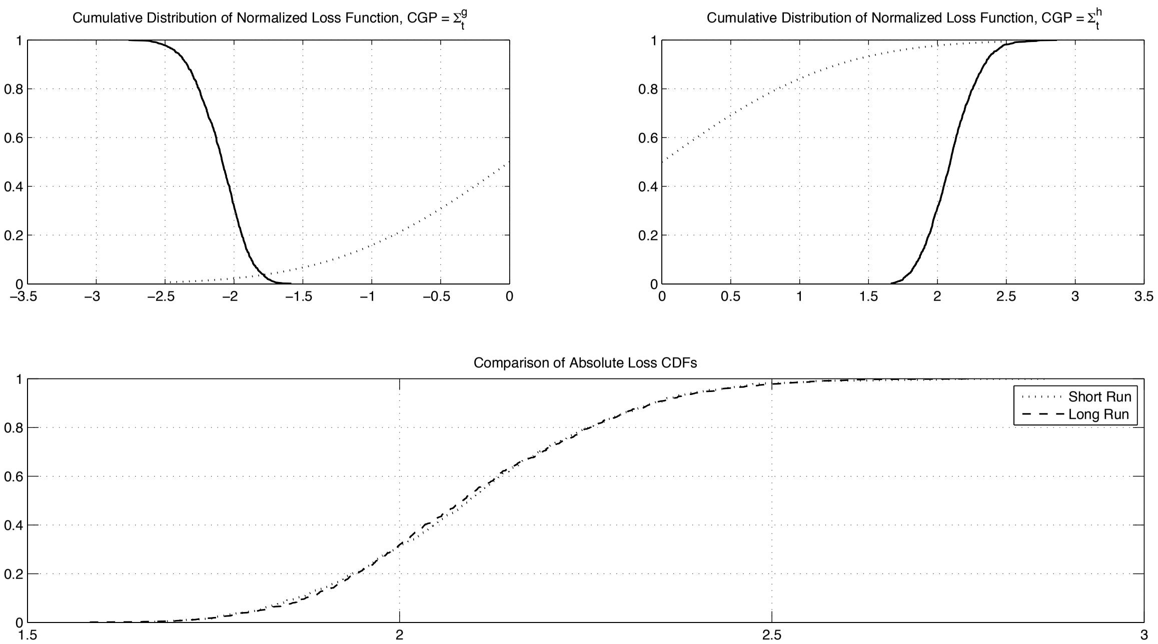


Figure D.14: The CDFs of the test statistic under various CGPs. The top left plot presents the CDF of the test statistic when the CGP is short run stable. The top right presents the CDF of the test statistic when the CGP is long run stable. The dotted line straddling the two is when neither CGP is correct and the true CGP is a random draw from a Wishart matrix distribution with an identity generating matrix and 24 degrees of freedom.



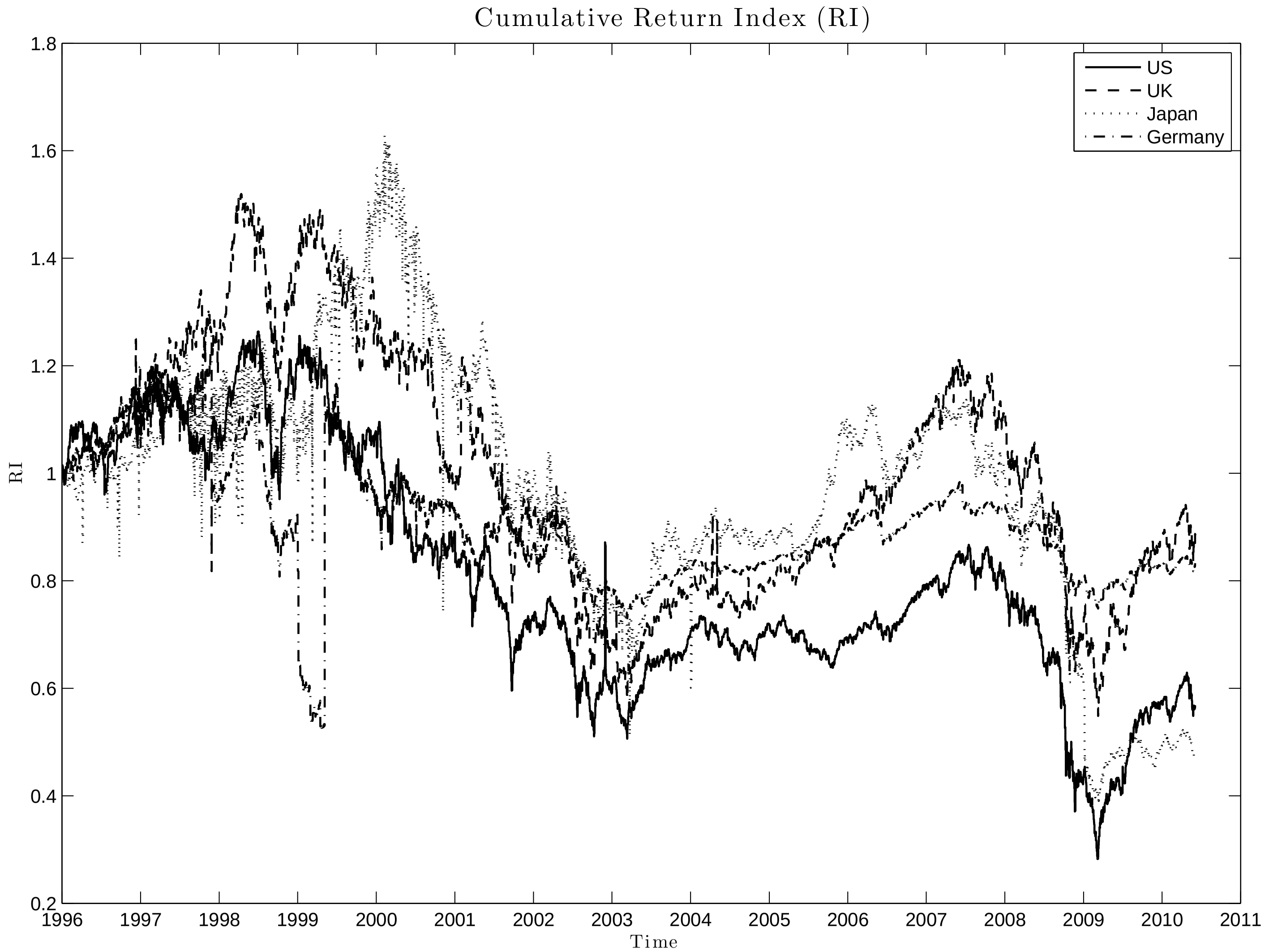


Figure D.15: Returns of Equally Weighted Portfolios of Constituent Stocks by Market. The minute by minute return index for an equally weighted portfolio of the constituent stocks generated by the filtered minute grid data.

Table D.2: List of Reuters Information Codes (RICs) for selected stocks, from each market.

The Reuters Tick History data service provides high frequency transacted price (Trade) data on all listed stocks in major markets and asset classes. The transacted prices are denoted in the local currency for each market.The selection criteria for stocks is as follows. First all active stocks traded over the sample period from January 1, 1996 to June 7, 2010 are listed. This is a function of availability as the Tick History data service begins collecting data from January 1, 1996. Second, the remaining stocks are ranked by total number of updated ticks and eliminate stocks with more than 1% of days with less informative ticks than minutes. Therefore the sample is constructed using the most actively traded stocks.

The primary goal of this study is to evaluate the covariance stability of actively traded assets and for computational tractability and to reduce the sample to a smaller cross section of only the most actively traded stocks for the US, UK, Japanese and German markets. Stocks, which have become bankrupt or undergone significant M&A activity over the sample are eliminated. The rational for this is that the objective of this study is not to compute the overall risk exposure in the market, but to evaluate the dynamics of the variance/covariance matrix of actively traded stocks.

For the German Equity data, most of the sample is sourced from Thompson-Reuters Tick History, however there is a missing period of data from January 1, 1998 to November 1, 2002, the missing ticks for this period the data is sourced directly from the exchange.

US		UK		Japan		Germany	
RIC	Name	RIC	Name	RIC	Name	RIC	Name
AA.N	ALCOA INC	AAL.L	ANGLO AMERICAN	4502.T	TAKEDA PHARM	ADSG.DE	ADIDAS
AIG.N	AMER INTL GROUP	BARC.L	BARCLAYS	4901.T	FUJIFILM HOLDING	ALVG.DE	ALLIANZ SE
AXP.N	AMER EXPRESS CO	BAY.L	BRITISH AIRWAYS	5108.T	BRIDGESTONE CORP	BASF.DE	BASF SE
BA.N	BOEING CO	BP.L	BR PETROLEUM	6501.T	HITACHI	BMWG.DE	BAY MOT WERKE
C.N	CITIGROUP	BSY.L	B SKY B	6701.T	NEC CORPORATION	CBKG.DE	COMMERZBANK
CAT.N	CATERPILLAR INC	BT.L	BRITISH TELECOM	6702.T	FUJITSU LTD	DTEGn.DE	DT TELEKOM N
DD.N	DU PONT CO	CPG.L	COMPASS GROUP	6752.T	PANASONIC	FMEG.DE	FRESEN MED CARE
DIS.N	WALT DISNEY CO	CW.L	CABLE & WIRELESS	6758.T	SONY CORP	HNKGp.DE	HENKEL AG
GE.N	GENERAL ELEC CO	GKN.L	GKN	6902.T	NIPPONDENSO	KARG.DE	KARSTADT
HD.N	HOME DEPOT INC	HSBA.L	HSBC	6963.T	ROHM CO LTD	AROG.DE	ARCANDOR AG
HON.N	HONEYWELL	III.L	3I GROUP	7203.T	TOYOTA MOTOR CO	LHAG.DE	DT LUFTHANSA AG
IBM.N	INTL BUS MACHINE	LAND.L	LAND SECURITIES	7267.T	HONDA MOTOR	VOWG.DE	VOLKSWAGEN AG
INTC.O	INTEL CORP	LGEN.L	LEGAL&GENERAL	9437.T	NTT DOCOMO	LING.DE	LINDE AG/LINDE
JNJ.N	JOHNSON&JOHNSON	MKS.L	MARKS&SPENCER	7751.T	CANON INC	MANG.DE	MAN AG/MAN SE
JPM.N	J P MORGAN	PRU.L	PRUDENTIAL	9501.T	TOKYO ELEC PWR	MEOG.DE	METRO AG
KO.N	COCA-COLA CO	PSON.L	PEARSON	7974.T	NINTENDO	MUVGn.DE	MUENCH. RUECK N
MCD.N	MCDONALDS CORP	RTO.L	RENTOKIL	9503.T	KANSAI ELE PWR	PRSG.DE	PREUSSAG AG
MMM.N	3M COMPANY	SBRY.L	SAINSBURY J	9984.T	SOFTBANK	RWEG.D	RWE ST/RWE ST A/RWE AG
MO.N	PHILIP MORRIS/ALTRIA	SGE.L	SAGE GROUP	8604.T	NOMURA HOLDINGS	SIEGn.DE	SIEMENS N
MRK.N	MERCK&CO	STAN.L	STANDRD CHARTER	9020.T	EAST JAPAN RY	TKAG.DE	THYSSEN KRUPP
MSFT.O	MICROSOFT CP	SVT.L	SEVERN TRENT	9432.T	NTT		
PFE.N	PFIZER INC	TSCO.L	TESCO	6502.T	TOSHIBA CORP		
PG.N	PROCTER&GAMBLE	ULVR.L	UNILEVER				
T.N	AT&T	VOD.L	VODAFONE				
UTX.N	UNITED TECH CP	WPP.L	WPP GROUP				
WMT.N	WAL-MART STORES						
XON.N	EXXON CP						

Figure D.16: Example of the data sorting and grid mapping.
The data here is for Quarter 3 of 2009. This quarter begins Wednesday, July 1, 2009 and runs to Wednesday, July 30, 2009. Plot (1,1) illustrates the data for the first Monday of July, July 6, 2009. The vertical dotted lines denote the break period, when the Japanese market ceases to trade. All transaction prices in this window are ignored and the returns from the last and first minute either side of the break are also ignored. Plot (1,2) illustrates the first week of July, running from July 6, 2009 to July 10, 2009. Plot (2,1) illustrates the set of 22 days data for July 2009. On Monday, 20, July 2009 the stock is not traded (this is the 14th day of data on the plot), no data for this day is recorded and this day is excluded from the covariance test statistic calculation. If the day is a public holiday then the day is completely excluded, however some days a stock is either thinly traded or not traded at at all, however the market is open. These days are completely excluded from the computation of the covariance equality test statistic, however these days are recorded as a zero contribution to the rolling loss tests, by restricting the column and row of the realised covariance matrix and its forecasts to vectors of zeros. Plot (2,2) shows the entire quarter of data, 3 more non trading days are obvious in the 8th 7th and 6th from last days, again the data for these days is excluded. For each stock a list of non-traded days and other relevant corporate actions (such as stock splits and repurchases) are compiled.

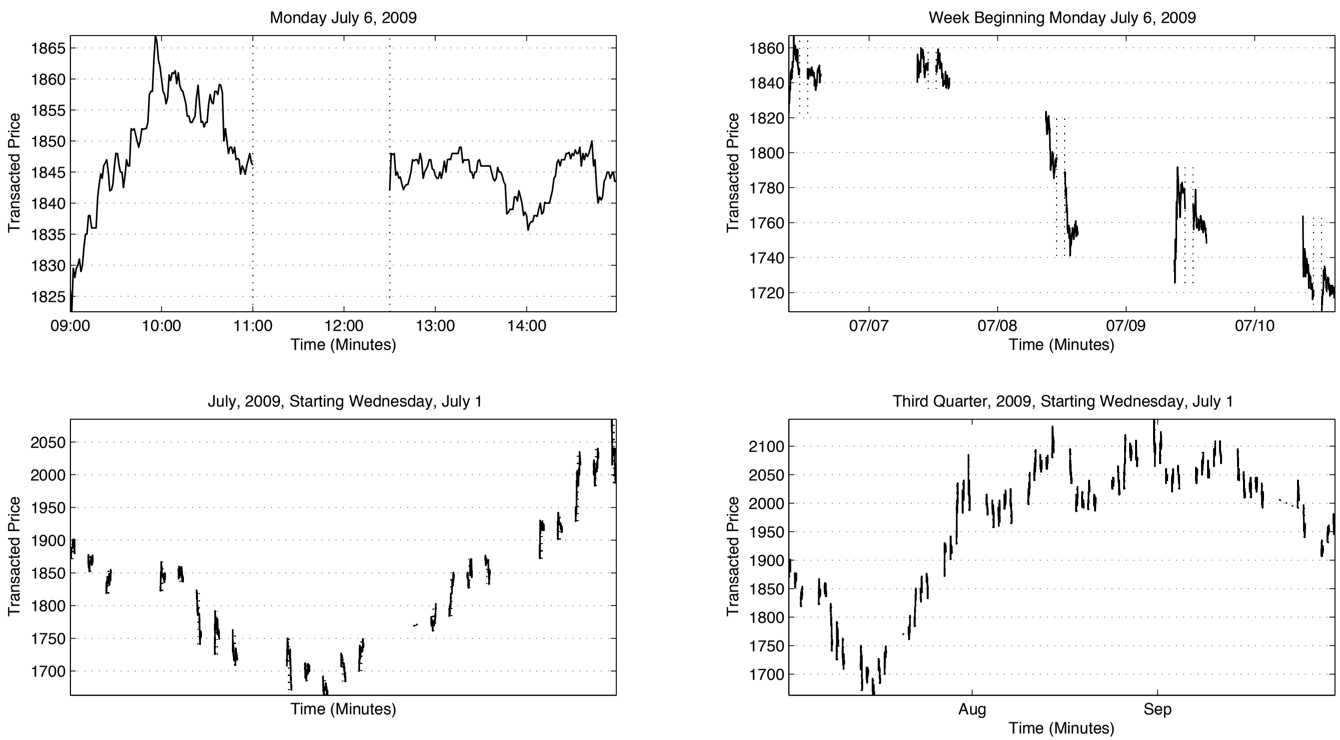


Figure D.17: Example: The Average Correlation Dynamics for the UK Market.

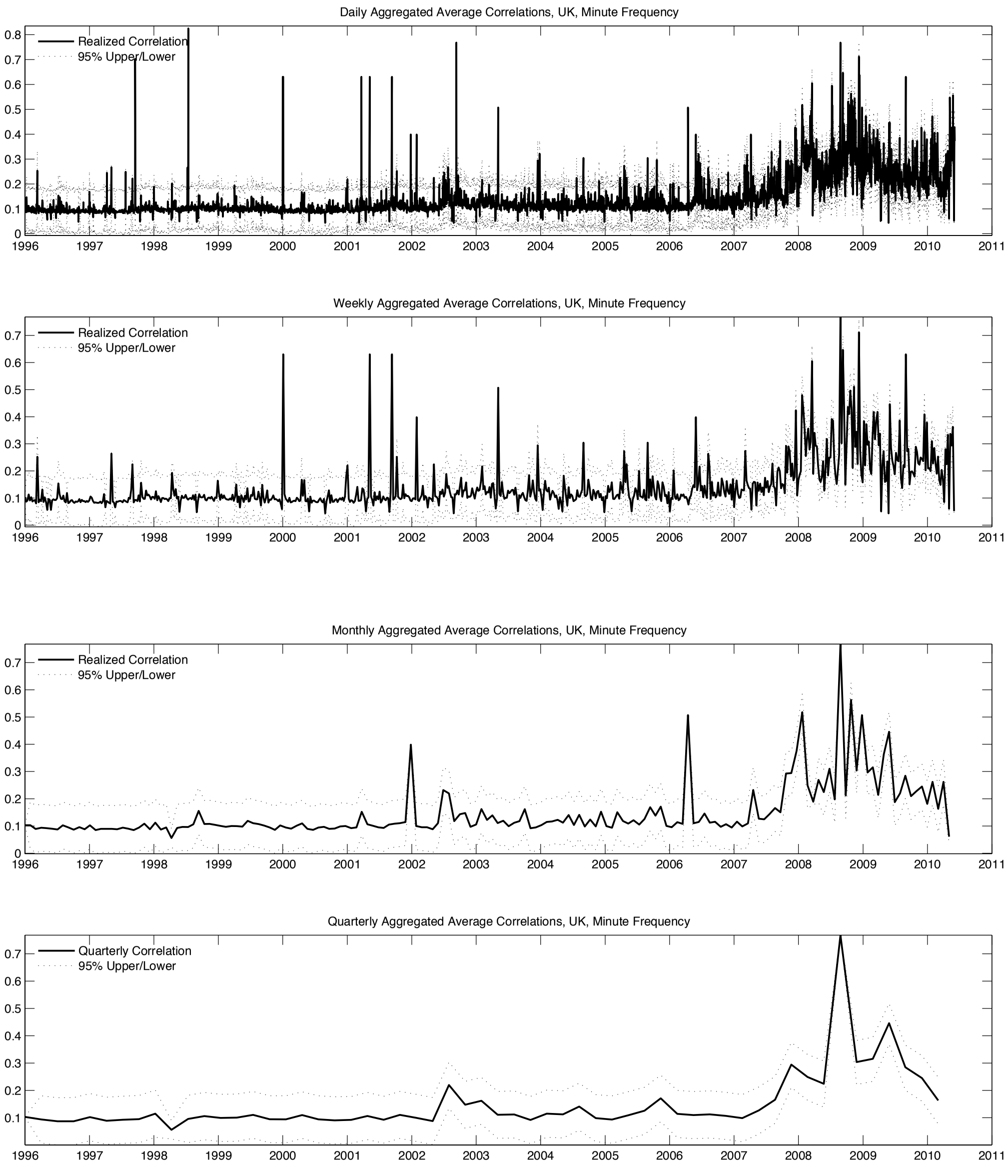


Table D.3: Stock Exchange Trading Times.

Notes: Thomson Reuters Tick History provides time stamps in GMT and an exchange local time offset (e.g. +1 for Daylight Saving in the UK, or -8 for Japan). Tick dates and times are converted to a decimal index using the `datenum` utility in MatLab (dates are all relative to January 1, 0000) and add the offset divided by 25 to convert to the exchange time. Therefore only one standardised minute grid is needed with the exchange hours above included, this avoids mistakes such as changes in daylight saving. It is also possible to use the universal time to compute intra-day correlations between stocks at points of overlap, for instance for the UK and US exchanges.

*For the FSE floor trading is continuous until 20:00, The sample is truncated at 17.30 as trading volumes thin out dramatically until the closing call auction.

Stock Exchange	Times	Holidays
NYSE	09:30 to 16:00 ET	Between 8 and 9 holidays per year
LSE	08:00 to 16:30 BST	8 holidays per year, subject to yearly review
TSE	09:00 to 11:00 and 12:30 to 15:00 JST	15 to 16 holidays per year, subject to review
FSE	09:00 to 20:00* CET	5 Holidays per year, subject to review

Table D.4: Proposes a variety of possible CGPs that could be estimated utilising the Wishart density and realized covariance analysis in the style of those outlined in this paper. Here $Q \sim \mathcal{MN}_{p,n}(0, \Xi, \Omega)$ is a draw from a matrix normal density, with p rows and n columns, with row-wise covariance Ξ and column-wise covariance Ω the obvious choices to reduce estimation issues is to use set Ξ and Ω as appropriately sized identity matrixes and to control the variation in Σ by adjusting the rows of the random matrix p.

Data Generating Process	Parameters (θ) /Bandwidth Choices	Comments
$\Sigma_t = m_{t-1}^{-1} {X'}_{t-1} X_{t-1}$	Choice of m	Stepwise Covariance Process short run stable
$\Sigma_t = h_i \sum_{i=1}^{p} m_{t-i}^{-1} {X'}_{t-i} X_{t-i}, \quad \sum_{i=1}^{p} h_i = 1$	Choice of m, functional form of h_i, e.g. $h_i = \frac{1}{p}$	Long run stable process alternative lag structures linear and exponentially weighted
$\Sigma_t = \psi_t \Sigma_1 + (1 - \psi_t) \Sigma_2$	$\theta = (\Sigma_1, \Sigma_2)$, Markov Transition Matrix $P(\psi)$	Simple Markov switching linear mixture model
$\Sigma_t = \psi \Sigma_0 + (1 - \psi) Q'Q$, $Q \sim \mathcal{MN}_{p,n}(0, \Xi, \Omega)$	$\theta = (\psi, \Sigma_0, p, \Xi, \Omega)$	Wishart stochastic covariance model with long run static covariance matrix
$\Sigma_t = \psi \left(m_{t-1}^{-1} {X'}_{t-1} X_{t-1} \right) + (1 - \psi) Q'Q$, $Q \sim \mathcal{MN}_{p,n}(0, \Xi, \Omega)$	$\theta = (\psi, p, \Xi, \Omega)$	Wishart stochastic covariance model with short run stepwise covariance matrix
$\Sigma_t = K_t K_t'$, $vechK_t = \Pi vechK_{t-1} + c + v_t$, $v_t \sim \mathcal{N}(0, \Omega)$	$\theta = (\Pi, c, \Omega)$	*vech* stochastic covariance model

Table D.5: In sample matrix equality test for US stocks.

The in sample equality tests are presented in Tables D.5 to D.8. The tables are in blocks of four columns, with the test statistic and critical statistics at the 90%, 95% and 99% significance. Each of the critical statistics are divided by a constant (10e6) following the method suggested in Muirhead (1983). For each year there is a slightly different critical statistic depending on how many days and stocks are included in the sample.

	Daily				Weekly				Monthly				Quarterly			
	Λ^*	$\alpha = 90\%$	$\alpha = 95\%$	$\alpha = 99\%$	Λ^*	$\alpha = 90\%$	$\alpha = 95\%$	$\alpha = 99\%$	Λ^*	$\alpha = 90\%$	$\alpha = 95\%$	$\alpha = 99\%$	Λ^*	$\alpha = 90\%$	$\alpha = 95\%$	$\alpha = 99\%$
1996	0.93016***	0.92160	0.92316	0.92609	0.26242***	0.18497	0.18567	0.18699	0.08518***	0.03974	0.04006	0.04068	0.03871***	0.01112	0.01129	0.01162
1997	0.93239***	0.91807	0.91963	0.92256	0.26315***	0.18497	0.18567	0.18699	0.08434***	0.03974	0.04006	0.04068	0.03924***	0.01112	0.01129	0.01162
1998	0.92818***	0.91807	0.91963	0.92256	0.26352***	0.18497	0.18567	0.18699	0.08554***	0.03974	0.04006	0.04068	0.03904***	0.01112	0.01129	0.01162
1999	0.92803***	0.91807	0.91963	0.92256	0.26062***	0.18497	0.18567	0.18699	0.08469***	0.03974	0.04006	0.04068	0.03910***	0.01112	0.01129	0.01162
2000	0.92603***	0.91455	0.91611	0.91903	0.25904***	0.18143	0.18213	0.18344	0.08514***	0.03974	0.04006	0.04068	0.03879***	0.01112	0.01129	0.01162
2001	0.92963***	0.91807	0.91963	0.92256	0.26428***	0.18497	0.18567	0.18699	0.08614***	0.03974	0.04006	0.04068	0.03918***	0.01112	0.01129	0.01162
2002	0.92656***	0.91807	0.91963	0.92256	0.26081***	0.18497	0.18567	0.18699	0.08479***	0.03974	0.04006	0.04068	0.03884***	0.01112	0.01129	0.01162
2003	0.92398***	0.91807	0.91963	0.92256	0.25870***	0.18497	0.18567	0.18699	0.08409***	0.03974	0.04006	0.04068	0.03818***	0.01112	0.01129	0.01162
2004	0.92814***	0.92160	0.92316	0.92609	0.25834***	0.18497	0.18567	0.18699	0.08367***	0.03974	0.04006	0.04068	0.03756***	0.01112	0.01129	0.01162
2005	0.91996***	0.91455	0.91611	0.91903	0.25308***	0.18143	0.18213	0.18344	0.08292***	0.03974	0.04006	0.04068	0.03752***	0.01112	0.01129	0.01162
2006	0.91922***	0.91455	0.91611	0.91903	0.25327***	0.18143	0.18213	0.18344	0.08310***	0.03974	0.04006	0.04068	0.03735***	0.01112	0.01129	0.01162
2007	0.92312***	0.91807	0.91963	0.92256	0.25715***	0.18497	0.18567	0.18699	0.08355***	0.03974	0.04006	0.04068	0.03766***	0.01112	0.01129	0.01162
2008	0.93116***	0.92160	0.92316	0.92609	0.26161***	0.18497	0.18567	0.18699	0.08595***	0.03974	0.04006	0.04068	0.04041***	0.01112	0.01129	0.01162
2009	0.92561***	0.91807	0.91963	0.92256	0.25945***	0.18497	0.18567	0.18699	0.08501***	0.03974	0.04006	0.04068	0.03896***	0.01112	0.01129	0.01162
2010	0.39729***	0.39319	0.39421	0.39613	0.11629***	0.07881	0.07927	0.08014	0.04161***	0.01831	0.01853	0.01895	0.01653***	0.00385	0.00395	0.00415
Whole	13.31802	13.23598	13.24190	13.25300	3.02858***	2.65235	2.65500	2.65997	0.89217***	0.60465	0.60592	0.60830	0.38006***	0.20263	0.20337	0.20475

Table D.6: In sample matrix equality test for UK stocks.

	Daily				Weekly				Monthly				Quarterly			
	Λ^*	$\alpha = 90\%$	$\alpha = 95\%$	$\alpha = 99\%$	Λ^*	$\alpha = 90\%$	$\alpha = 95\%$	$\alpha = 99\%$	Λ^*	$\alpha = 90\%$	$\alpha = 95\%$	$\alpha = 99\%$	Λ^*	$\alpha = 90\%$	$\alpha = 95\%$	$\alpha = 99\%$
1996	0.80017***	0.78807	0.78952	0.79223	0.20801***	0.15826	0.15891	0.16013	0.06760***	0.03404	0.03434	0.03491	0.02938***	0.00954	0.00970	0.01001
1997	0.79348***	0.78506	0.78650	0.78921	0.20524***	0.15826	0.15891	0.16013	0.06441***	0.03404	0.03434	0.03491	0.02700***	0.00954	0.00970	0.01001
1998	0.79371***	0.78506	0.78650	0.78921	0.21292***	0.15826	0.15891	0.16013	0.06475***	0.03404	0.03434	0.03491	0.02768***	0.00954	0.00970	0.01001
1999	0.79118***	0.78506	0.78650	0.78921	0.21432***	0.15826	0.15891	0.16013	0.06546***	0.03404	0.03434	0.03491	0.02834***	0.00954	0.00970	0.01001
2000	0.78710***	0.78205	0.78349	0.78620	0.20510***	0.15524	0.15588	0.15709	0.06574***	0.03404	0.03434	0.03491	0.02856***	0.00954	0.00970	0.01001
2001	0.79176***	0.78506	0.78650	0.78921	0.20928***	0.15826	0.15891	0.16013	0.06603***	0.03404	0.03434	0.03491	0.02806***	0.00954	0.00970	0.01001
2002	0.79335***	0.78506	0.78650	0.78921	0.20810***	0.15826	0.15891	0.16013	0.06493***	0.03404	0.03434	0.03491	0.02786***	0.00954	0.00970	0.01001
2003	0.79209***	0.78506	0.78650	0.78921	0.20949***	0.15826	0.15891	0.16013	0.06603***	0.03404	0.03434	0.03491	0.02834***	0.00954	0.00970	0.01001
2004	0.79487***	0.78807	0.78952	0.79223	0.20958***	0.15826	0.15891	0.16013	0.06586***	0.03404	0.03434	0.03491	0.02780***	0.00954	0.00970	0.01001
2005	0.78773***	0.78205	0.78349	0.78620	0.20725***	0.15524	0.15588	0.15709	0.06585***	0.03404	0.03434	0.03491	0.02813***	0.00954	0.00970	0.01001
2006	0.78824***	0.78205	0.78349	0.78620	0.20800***	0.15524	0.15588	0.15709	0.06651***	0.03404	0.03434	0.03491	0.02808***	0.00954	0.00970	0.01001
2007	0.79234***	0.78506	0.78650	0.78921	0.21419***	0.15826	0.15891	0.16013	0.06938***	0.03404	0.03434	0.03491	0.03073***	0.00954	0.00970	0.01001
2008	0.90405***	0.78807	0.78952	0.79223	0.17828***	0.15826	0.15891	0.16013	0.06601***	0.03404	0.03434	0.03491	0.02248***	0.00954	0.00970	0.01001
2009	0.55710	0.78506	0.78650	0.78921	0.68070***	0.15826	0.15891	0.16013	0.19032***	0.03404	0.03434	0.03491	0.05633***	0.00954	0.00970	0.01001
2010	0.34201***	0.33631	0.33725	0.33903	0.10267***	0.06747	0.06790	0.06870	0.03740***	0.01570	0.01591	0.01630	0.01767***	0.00331	0.00341	0.00359
Whole	11.32462***	11.31426	11.31973	11.32999	2.74423***	2.26761	2.27006	2.27466	0.78397***	0.51710	0.51828	0.52048	0.32370***	0.17337	0.17405	0.17533

Table D.7: In sample matrix equality test for Japanese stocks.

	Daily				Weekly				Monthly				Quarterly			
	Λ^*	$\alpha = 90\%$	$\alpha = 95\%$	$\alpha = 99\%$	Λ^*	$\alpha = 90\%$	$\alpha = 95\%$	$\alpha = 99\%$	Λ^*	$\alpha = 90\%$	$\alpha = 95\%$	$\alpha = 99\%$	Λ^*	$\alpha = 90\%$	$\alpha = 95\%$	$\alpha = 99\%$
1996	0.55897***	0.55234	0.55355	0.55583	0.14105***	0.11109	0.11164	0.11266	0.04260***	0.02397	0.02422	0.02471	0.01777***	0.00675	0.00689	0.00715
1997	0.55627***	0.55023	0.55144	0.55371	0.14836***	0.11109	0.11164	0.11266	0.04681***	0.02397	0.02422	0.02471	0.02192***	0.00675	0.00689	0.00715
1998	0.55528***	0.55023	0.55144	0.55371	0.15032***	0.11109	0.11164	0.11266	0.04498***	0.02397	0.02422	0.02471	0.01886***	0.00675	0.00689	0.00715
1999	0.55431***	0.55023	0.55144	0.55371	0.15152***	0.11109	0.11164	0.11266	0.04658***	0.02397	0.02422	0.02471	0.01951***	0.00675	0.00689	0.00715
2000	0.55271***	0.54813	0.54933	0.55160	0.14570***	0.10898	0.10951	0.11053	0.04877***	0.02397	0.02422	0.02471	0.02130***	0.00675	0.00689	0.00715
2001	0.55798***	0.55023	0.55144	0.55371	0.15228***	0.11109	0.11164	0.11266	0.04862***	0.02397	0.02422	0.02471	0.02145***	0.00675	0.00689	0.00715
2002	0.55489***	0.55023	0.55144	0.55371	0.15235***	0.11109	0.11164	0.11266	0.04819***	0.02397	0.02422	0.02471	0.02037***	0.00675	0.00689	0.00715
2003	0.55856***	0.55023	0.55144	0.55371	0.15318***	0.11109	0.11164	0.11266	0.04958***	0.02397	0.02422	0.02471	0.02312***	0.00675	0.00689	0.00715
2004	0.56495***	0.55234	0.55355	0.55583	0.17289***	0.11109	0.11164	0.11266	0.06798***	0.02397	0.02422	0.02471	0.05300***	0.00675	0.00689	0.00715
2005	0.56344***	0.54813	0.54933	0.55160	0.17375***	0.10898	0.10951	0.11053	0.07230***	0.02397	0.02422	0.02471	0.06124***	0.00675	0.00689	0.00715
2006	0.55900***	0.54813	0.54933	0.55160	0.16319***	0.10898	0.10951	0.11053	0.06253***	0.02397	0.02422	0.02471	0.04275***	0.00675	0.00689	0.00715
2007	0.56348***	0.55023	0.55144	0.55371	0.17170***	0.11109	0.11164	0.11266	0.06809***	0.02397	0.02422	0.02471	0.05206***	0.00675	0.00689	0.00715
2008	0.56758***	0.55234	0.55355	0.55583	0.17117***	0.11109	0.11164	0.11266	0.06350***	0.02397	0.02422	0.02471	0.04449***	0.00675	0.00689	0.00715
2009	0.56177***	0.55023	0.55144	0.55371	0.17086***	0.11109	0.11164	0.11266	0.06504***	0.02397	0.02422	0.02471	0.04599***	0.00675	0.00689	0.00715
2010	0.24131***	0.23587	0.23666	0.23815	0.07222***	0.04743	0.04779	0.04846	0.02663***	0.01109	0.01126	0.01159	0.01315***	0.00236	0.00244	0.00260
Whole	7.93286**	7.92262	7.92719	7.93578	1.91652***	1.58851	1.59056	1.59441	0.55087***	0.36253	0.36351	0.36536	0.22685***	0.12168	0.12225	0.12332

Table D.8: In sample matrix equality test for German stocks.

	Daily				Weekly				Monthly				Quarterly			
	Λ^*	$\alpha = 90\%$	$\alpha = 95\%$	$\alpha = 99\%$	Λ^*	$\alpha = 90\%$	$\alpha = 95\%$	$\alpha = 99\%$	Λ^*	$\alpha = 90\%$	$\alpha = 95\%$	$\alpha = 99\%$	Λ^*	$\alpha = 90\%$	$\alpha = 95\%$	$\alpha = 99\%$
1996	-	0.45014	0.45123	0.45329	-	0.09063	0.09112	0.09205	-	0.01960	0.01983	0.02026	-	0.00554	0.00566	0.00590
1997	0.45022**	0.44842	0.44951	0.45156	0.12671***	0.09063	0.09112	0.09205	0.04239***	0.01960	0.01983	0.02026	0.02012***	0.00554	0.00566	0.00590
1998	0.45539***	0.44842	0.44951	0.45156	0.13099***	0.09063	0.09112	0.09205	0.04276***	0.01960	0.01983	0.02026	0.01963***	0.00554	0.00566	0.00590
1999	0.46273***	0.44842	0.44951	0.45156	0.11575***	0.09063	0.09112	0.09205	0.03686***	0.01960	0.01983	0.02026	0.01566***	0.00554	0.00566	0.00590
2000	0.44670*	0.44670	0.44779	0.44984	0.12030***	0.08890	0.08939	0.09031	0.03899***	0.01960	0.01983	0.02026	0.01724***	0.00554	0.00566	0.00590
2001	0.44842*	0.44842	0.44951	0.45156	0.12202***	0.09063	0.09112	0.09205	0.03910***	0.01960	0.01983	0.02026	0.01734***	0.00554	0.00566	0.00590
2002	0.46527***	0.44842	0.44951	0.45156	0.12537***	0.09063	0.09112	0.09205	0.04025***	0.01960	0.01983	0.02026	0.02709***	0.00554	0.00566	0.00590
2003	0.45524***	0.44842	0.44951	0.45156	0.13013***	0.09063	0.09112	0.09205	0.04290***	0.01960	0.01983	0.02026	0.01969***	0.00554	0.00566	0.00590
2004	0.45421***	0.45014	0.45123	0.45329	0.12692***	0.09063	0.09112	0.09205	0.04144***	0.01960	0.01983	0.02026	0.01890***	0.00554	0.00566	0.00590
2005	0.44903**	0.44670	0.44779	0.44984	0.12357***	0.08890	0.08939	0.09031	0.04076***	0.01960	0.01983	0.02026	0.01844***	0.00554	0.00566	0.00590
2006	0.45088***	0.44670	0.44779	0.44984	0.12462***	0.08890	0.08939	0.09031	0.04110***	0.01960	0.01983	0.02026	0.01882***	0.00554	0.00566	0.00590
2007	0.45412***	0.44842	0.44951	0.45156	0.12775***	0.09063	0.09112	0.09205	0.04167***	0.01960	0.01983	0.02026	0.01893***	0.00554	0.00566	0.00590
2008	0.45894***	0.45014	0.45123	0.45329	0.13036***	0.09063	0.09112	0.09205	0.04282***	0.01960	0.01983	0.02026	0.02026***	0.00554	0.00566	0.00590
2009	0.45416***	0.44842	0.44951	0.45156	0.12827***	0.09063	0.09112	0.09205	0.04223***	0.01960	0.01983	0.02026	0.01972***	0.00554	0.00566	0.00590
2010	0.19483***	0.19231	0.19302	0.19437	0.05719***	0.03873	0.03905	0.03966	0.02065***	0.00908	0.00924	0.00954	0.00838***	0.00195	0.00202	0.00216
Whole	6.16163	6.45269	6.45682	6.46457	1.22392	1.29413	1.29598	1.29946	0.22702	0.29551	0.29639	0.29806	0.06656	0.09926	0.09977	0.10074